OUTLINES OF A METHOD FOR DETERMINING THE GEOID IN SWEDEN BY FREE-AIR ANOMALIES

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ERIK TENGSTRÖM

Uppsala 1954

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Bibliography (Bibl.)

Atlases and maps.

1. Bonniers stora världsatlas, Stockholm 1951.
2. Höjdkartan över Sverige i skalan 1: 500 000.
3. French marine charts,
   a) S.H.M., Paris 1921: Mers d'Europe (de France en Islande), echelle 1: 2 986 700.
   b) S.H.M., 1912: Océan Atlantique Nord et Mers Boréale, echelle 1: 6 600 000.

Lectures and monographs.

4. *Contini, C.:* Il calcolo dell'influenza topografica. (Bolletino di Geodesia, XII: 1), 1953.
5. Coast and Geodetic Survey: Principle triangulation of Sweden European Datum 1951.
6. *De Graaff-Hunter, J.:* Geodetic uses of gravity measurements and their appropriate reduction. Proc. Roy Soc, A, vol 206, 1951.
7. *Heiskanen, W.:* Schwerkraft und isostatische Kompensation in Norwegen, Helsinki 1926.
8. — Catalogue of the isostatically reduced gravity stations, Helsinki 1939.
9. *Hirvonen, R.:* On the precision obtainable for gravimetrical determinations of the geoid. (Map Chart Res Lab, Ohio), 1953.
10. *Jeffreys, H.:* The determination of the earth's gravitational field (two papers in Month Not, Geoph Suppl., Vol 5, N° 1), 1941.
11. *Lambert, W. D.:* The Comparisons of deflections observed astronomically with those observed gravimetrically, 1950 (contents given by Rice, see bibl. 16).
12. *Ledersteger, K.:* Die absolute Lotabweichung in Potsdam ..., Bull Géod 1952.
13. *Meinez, Vening:* New formulas for systems of deflections..., Bull Géod N° 15, 1950.
14. *Pavel, Bonsdorff:* Bestimmung der Längenunterschiede der Längencentralen, B.G.K., Helsinki 1934.
15. *Pesonen, U.:* Geodetic operations in Finland, Helsinki 1951.
16. *Rice, D. A.:* Deflections of the vertical from gravity anomalies, Bull Géod 1952.
17. *Rudzky, M. P.:* Sur la détermination de la figure de la terre d'apres les mesures de la gravite, Bull. astron. 22, 1905.
18. *Sollins, A. D.:* Tables for the computation of deflections of the vertical from gravity anomalies, Bull Géod 1946.
19. *Tanni, L.:* On the contimental undulations of the geoid as determined from the present gravity material, Helsinki 1948 (Isost. Inst.).
20. *Wideland, B.:* Ausgleichung des astronomisch-geodätischen Dreiecksnetzes Signilskär-Omberg. Medd. från RAK nr 1, Stockholm 1943.
21. — Relative Schweremessungen in Süd- und Mittelschweden in den Jahren 1943—44, Medd. från från RAK nr 6, Stockholm 1946.
22. — Relative gravity measurements in Middle and North Sweden 1945—48. Medd. från RAK nr 14, Stockholm 1951.
23. — Deflections of the vertical of some Swedish Laplace-stations gravimetrically determined, Medd. från RAK nr 21, Stockholm 1954.
24. *Wolf, H.:* Geoidvergleiche und absolute Lotabweichungen, Deutsche Geod Komm Reihe A, 5, 1952.

Preface.

This investigation was started in 1952 at the proposal of Professor *G. A. Rune*, former Head of the Geodetic Department at the Geographical Survey Office of Sweden. For the kind interest he has shown in all problems related with the investigation I wish to express my great gratitude.

The Director-in-chief of the Geographical Survey Office of Sweden, *V. Hernlund* and Professor *L. Asplund*, Head of the Geodetic Department I owe a great debt of gratitude for their kindness to give me the necessary free time for finishing my work.

My wife has spent the main part of her time to help me with tiring computations. I thank her for that. And I will never forget that her encouragement has been the main cause for my surmounting all difficulties in connection with the work.

All others who have helped me or being interested in the subject I owe a debt of gratitude.

Introduction.

The determination of geoidal undulations over restricted areas has hitherto ordinarily been carried out by the aid of astronomical observations for latitude and longitude. The method is accurate with regard to the separate deflections, but to utilize fully this accuracy it would be necessary to have a very dense net of astronomical stations. And the corresponding observation work would be overwhelming. Usually the station density aimed at is only about 1 p./900 sqkms, and the geoidal profiles are constructed under the presumption that the curvature between two consecutive stations has a constant value.

I am of the opinion that the astronomical observations, as they are carried out now and have been carried out during centuries (that is in the form of absolute determinations of long and lat), are not the most suitable ones for determining the geoidal details. In recent time two lines of thought concerning this problem of geodesy have been discussed in the literature and at the congress-meetings: *the geometrical method of measuring vertical-angles* and *the gravimetrical method of calculating curvatures from Stokes' formula.* In both cases the astronomical deflection points are intended to play the role of control-stations forming a net with relatively low density. From these stations the local geoidal structure should be studied by means of one of the aforesaid methods. The vertical-angle method has been tested in mountain-regions where the refraction is small and steady. Here the observational results are satisfactory, giving deflection-differences with an accuracy of about $\pm$ 1". In order to utilize the method successfully during ordinary weatherconditions it would be necessary to eliminate the adverse effects of fast and irregular variations of refraction. This could perhaps be done with an apparatus similar to Michelson's double-slit-interferometer, utilized in measuring the angle of dispersion between two different wave-lengths from the same light-signal. Some experiments with such an apparatus in monochromatic light have been carried out at the Institute of Technology at Stockholm in order to test the accuracy in defining the disappearing of interferencefringes, and it has been found that the mean error when measuring an angle between two light-sources of about 10" at 600 mts distance will be less than $\pm$ 0."2 after 10 readings. The difficulty which arises when different wave-lengths shall be compared chiefly depends upon the insufficient achromatisme in ordinary telescope-lenses. With the aid of an »apochromatic» lens, however, it will probably be possible to master the problem. The experiments are continued.

The method for studies of geoidal structure with the aid of gravimetrical curvatures shall be investigated in this paper. It will be seen that the accuracy is great enough when we study a somewhat generalized gravity field, which may be defined from the

given density of gravimetrical stations. The corresponding generalized geoid within our area of investigation seems to agree well with the isostatic cogeoid (according to Airy—Heiskanen).

The astronomic-geodetic deflections are intended to control and orientate the gravimetrically determined geoidal surface. For that purpose they have to be reduced for the difference between the real geoidal field and the generalized one (ch VI).

In ch VII an attempt has been made to obtain an *absolute* orientation (in ξ and η) for one of the deflection stations (Östra Älvhöjden). The result is in good agreement with the recent values of gravimetrical deflections at the Helmertturm in Potsdam.

I.

Reductions of gravity and astronomical observations down to the cogeoid.

We assume that the gravity is known everywhere on a level surface of the earth's gravitational field. If the surface comprises the whole mass of the earth, it is possible to determine it by Stokes' formula. This formula follows from *the outer space solution* of Neumann's theorem in the potential theory, and it is correct at least to the second degree of the flattening of the earth (: errors due to the derivation of the formula are of the order $\alpha^3 R$, where α is the flattening, R the earth's mean radius).

But if we have to compute the geoid, the problem—due to the projecting masses—is no longer an outer space one. This difficulty can, however, be overcome by making an imaginary transport of the projecting masses to the inner space of the geoid. In general, such a transport must displace and deform all the level surfaces including the geoid itself. We usually describe the new field as a *cogeoidal* field and the new »geoid» as a *cogeoid*. According to the manner in which the masstransport is performed, we get different *cogeoids*, which may differ more or less from the geoid. Among all these, *Rudzki's cogeoid* (bibl. 17) is the only one which coincides with the geoid. If we reduce all gravity observations according to Rudzki's method of inversion, Stokes' formula then gives the geoid directly. In practice we obtain the same result when we make a pure reflection of the topography down below the geoid without altering the density of the reflected matter. It can be shown that the error resulting by using this approximation of Rudzki's method does not exceed about 3 mms in geoid elevation at any point of the world, and this only amounts to 1 p. c. of the errors due to the uncertainty in the derivation of Stokes' formula.

The reductions of gravity and astronomical observations in this paper are intended to give relations between observed quantities and their counterparts on this special cogeoid, which coincides with the geoid but the gravity distribution of which need not be the same as that of the geoid.

As regards the accuracy of the formulas for reduction, we shall consider it satisfactory if the gravimetrical reductions—when necessary—can be calculated with an error, which does not exceed 1 mgal, and if we can compute all astronomical reductions with an error $< 0.''5$. This statement is due to the fact that we can only estimate, but cannot definitely determine, those parts of the reductions which are due to irregularities in the massdistribution within the topography and below the geoid.

The investigations on the problem of gravimetrical and astronomical reductions to the geoid are very extensively treated in geodetic literature. In the following pages, I have accepted the main lines of thought evolved by de Graaff—Hunter (bibl. 6) and W. D. Lambert (bibl. 11) in recent publications.

Reductions of gravity values.

In order to obtain the gravity difference between the cogeoid and the theoretical earth, that is, the anomaly $\triangle g$ on the cogeoid, we must take the process of mass-transport into consideration. If g_{obs} is the observed gravity at a station P above the geoid, we obtain $\triangle g$ after removing all topography (to infinity), projecting the station to the geoid at P_0 and placing the reflected topography below the geoid:

$$\triangle g = g_{obs} - \triangle g_t + \triangle g_f + \triangle g_i - \gamma \quad \text{..... I: 1}$$

Here $-\triangle g_t$ is the effect of removing all topography above sea level, $+\triangle g_f$ is the free air reduction from P to P_0 in the field of the remaining masses, $+\triangle g_i$ is the effect of reflecting the topography and γ is the corresponding gravity on the theoretical earth (according to the international formula of gravity).

In computing $-\triangle g_t + \triangle g_i$ we can use the table I (pp 12,13).*) Reckoning in the plane, we have *the effect of a topographic zone* between the radii b_1 *kms* and b_2 *kms* with the mean height h *kms* and the mean density ϱ:

$$\delta \triangle g_t = -2\pi k^2 \varrho \left(b_2 - b_1 - \sqrt{b_2^2 + h^2} + \sqrt{b_1^2 + h^2}\right) \quad \text{..... I: 2}$$

if P is situated on the geoid (P_0).

We have in I: 2 $k^2 = 6.66$ ($k^2 . 10^{-8}$ *cgs* is the constant of universal gravitation). For P_0 we can write $\delta \triangle g_t = -k_t \cdot \varrho$ and for P (if $h_p = h$), $\delta \triangle g_t = +k_t \varrho$. The table now gives k_t for different zones and mean heights between the radii 0 km and 5 kms and the heights 20 mts and 500 mts. Beyond 5 kms we have, for a zone of width 1 *km*, the simple formula:

$$\delta \triangle g_t = -20.92 \varrho \frac{h^2}{b^2} = -k_t \cdot \varrho \quad \text{..... I: 2 a}$$

Here h is the mean height in *kms*, when b, the mean radius of the zone, is in kms. The formula is valid for $h_p = 0$. To obtain k_t for a zone in relation to a station with the heights h mts and h_p mts respectively, we have:

a) if $h_p > h$: take k_t for h_p and subtract

$$k_t \text{ for } h_p - h$$

b) if $h_p < h$: take k_t for h_p and subtract

$$k_t \text{ for } h - h_p$$

The value of $\triangle g_t$ will be expressed in *mgals*. The range of heights in the table is chosen with respect to the topography within the investigated area (chs II, VI, VII).

The effect of the reflected topography $\delta \triangle g_i$ for the same zones within 5 kms can be taken from the same table. We have $\delta \triangle g_i = -k_t \cdot \varrho$ mgals, where k_t corresponds to the mean height h of the zone. Beyond $b = 5$ kms, we have as before $k_t = 20.92 \frac{h^2}{b^2}$, and the effect is $k_t \cdot \varrho$ mgals for a zone of width 1 km.

*) Recently C. Contini has published nomograms for the same computations (bibl. 4).

The total effect from a zone is then:

$$\delta(-\triangle g_t + \triangle g_i) = (\triangle k_t + k_t^{(\triangle h)})\,\varrho \quad \text{I: 3}$$

where $\triangle k_t$ is the difference between k_t for h and for h_p, $k_t^{(\triangle h)}$ is the value of k_t for $h_p - h$.

The formula in the plane is valid over a range of some 100 kms, if the the height in the table does not exceed 500 mts. When $h_p = 0$ and $h = 500$ mts, we find that the effect of all $\delta(-\triangle g_t + \triangle g_i)$ from $b = 20$ kms to the antipode is less than 1 mgal. Within the investigated area it is sufficient therefore, to calculate the reduction with a maximum radius of 20 kms.

In computing $\triangle g_f$ we can ordinarily use the gravity gradient *0.309 mgals/m* from the international gravity formula instead of the gradient in the field of the remaining masses (p 10). If the mean curvature of the corresponding level surfaces in the vicinity of the station does not exceed 5″/10 kms the error when using the normal gradient can not be greater than 0.5 p.c., that is for $h_p = 500$ mts (the reduction 155 mgals) < 1 mgal. In such cases we get $\triangle g_f$ within 1 mgal from the linear expression

$$\triangle g_f = 0.309\, h_p \text{ mgals} \quad \text{I: 4}$$

This accuracy of gravity reductions is of course not necessary for the outer regions in our field of integration (areas II and III of ch II). Here the errors due to using the normal gradient tend to cancel out in the average values of widespread stations within one compartment. Consequently it has been unnecessary here to correct free air anomalies taken from gravity catalogues even if one station or another may be suspected to have abnormally great curvatures in the level surfaces of the Bouguer anomaly field. For the area I (ch II) alone an attempt hasbeen made to estimate maximum curvatures for level surfaces of the Bouguer field. In dense gravity regions of this area, I have calculated mean gravimetrical curvatures in disks of about 5 kms radius round stations of great field curvature. These estimates show the existence of local curvatures greater than 20″/10 kms, which means an error of about 3 mgals in the free air correction if the station height is 500 mts. De Graaff–Hunter's proposal for observation of curvatures in the neighbourhood of gravimetrical deflection stations (bibl. 6) thus seems to be very important.

By calculating $-\triangle g_t + \triangle g_i$ an error results in the cogeoidal anomalies, depending upon the uncertainty of ϱ. Taking an extreme case ($h_p = 0$ mts; $h = 500$ mts), we have for the total reduction (from table I and formula I: 2 a) $+ 41\,\varrho$ mgals, that is an error of 4 mgals, if ϱ is known with an error of 10 p.c. In our areas II and III such errors for single points have no adverse effects. And in the area I the minuteness of the terrain corrections (usually < 1 mgal) indicates that this uncertainty should not be taken into account.

Table I

Rules for use see p 10.

zone h mts	0—25 mts	25—50	50—75	75— 100	100— 150	15— 200	200— 250	250— 300	300— 350	350— 400	400— 500	500— 600	600— 700
20	0.543	0.132	0.051	0.027	0.027	0.014	0.008	0.006	0.004	0.003	.0003	0.004	0.002
40	0.746	0.340	0.169	0.096	0.103	0.054	0.033	0.022	0.016	0.012	0.016	0.011	0.008
60	0.837	0.498	0.295	0.185	0.212	0.115	0.071	0.049	0.035	0.026	0.037	0.025	0.018
80	0.886	0.605	0.405	0.276	0.337	0.192	0.122	0.804	0.061	0.045	0.065	0.043	0.031
100	0.917	0.681	0.494	0.359	0.466	0.279	0.182	0.127	0.093	0.071	0.101	0.068	0.049
120	0.938	0.735	0.564	0.431	0.594	0.376	0.252	0.178	0.133	0.101	0.144	0.097	0.070
140	0.953	0.776	0.621	0.493	0.706	0.468	0.323	0.233	0.176	0.135	0.193	0.131	0.095
160	0.965	0.808	0.666	0.545	0.811	0.556	0.393	0.288	0.219	0.170	0.246	0.248	0.122
180	0.973	0.833	0.703	0.589	0.906	0.639	0.461	0.343	0.264	0.207	0.301	0.208	0.157
200	0.981	0.853	0.734	0.627	0.987	0.718	0.531	0.402	0.311	0.247	0.364	0.253	0.186
220	0.986	0.869	0.759	0.658	1.061	0.793	0.599	0.462	0.362	0.289	0.431	0.301	0.224
240	0.990	0.883	0.781	0.686	1.126	0.862	0.664	0.519	0.412	0.332	0.499	0.352	0.260
260	0.995	0.895	0.802	0.711	1.186	0.926	0.725	0.575	0.461	0.374	0.568	0.404	0.301
280	0.999	0.907	0.818	0.734	1.242	0.986	0.784	0.628	0.509	0.416	0.637	0.457	0.343
300	1.002	0.916	0.833	0.753	1.289	1.040	0.838	0.680	0.556	0.459	0.707	0.513	0.387
320	1.004	0.923	0.845	0.770	1.332	1.090	0.890	0.730	0.602	0.501	0.781	0.570	0.433
340	1.007	0.930	0.856	0.784	1.371	1.136	0.937	0.777	0.648	0.543	0.852	0.628	0.479
360	1.009	0.937	0.866	0.798	1.407	1.178	0.983	0.823	0.691	0.584	0.922	0.686	0.527
380	1.011	0.943	0.876	0.811	1.440	1.219	1.027	0.866	0.753	0.623	0.992	0.744	0.574
400	1.013	0.948	0.884	0.822	1.469	1.254	1.067	0.908	0.774	0.662	1.061	0.803	0.623
420	1.014	0.952	0.891	0.832	1.496	1.286	1.105	0.947	0.813	0.700	1.132	0.862	0.672
440	1.016	0.957	0.898	0.841	1.520	1.317	1.140	0.984	0.851	0.737	1.197	0.921	0.722
460	1.017	0.961	0.904	0.850	1.543	1.346	1.173	1.020	0.886	0.772	1.262	0.978	0.771
480	1.019	0.964	0.910	0.858	1.565	1.375	1.205	1.053	0.920	0.805	1.327	1.035	0.821
500	1.020	0.968	0.916	0.866	1.585	1.401	1.234	1.084	0.952	0.837	1.389	1.091	0.870

k_t

700— 800	800— 900	900— 1000	1000— 1200	1200— 1400	1400— 1600	1600— 1800	1800— 2000	2000— 2500	2500— 3000	3000— 3500	3500— 4000	4000— 5000	zone h mts
0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000	20
0.006	0.005	0.004	0.006	0.004	0.003	0.002	0.002	0.003	0.002	0.002	0.002	0.002	40
0.013	0.010	0.008	0.013	0.009	0.007	0.005	0.005	0.007	0.004	0.004	0.004	0.003	60
0.023	0.018	0.014	0.022	0.016	0.012	0.009	0.008	0.013	0.008	0.006	0.005	0.007	80
0.037	0.029	0.023	0.035	0.025	0.019	0.014	0.012	0.021	0.013	0.010	0.008	0.011	100
0.053	0.041	0.033	0.050	0.036	0.027	0.020	0.017	0.030	0.019	0.014	0.011	0.015	120
0.072	0.056	0.045	0.049	0.037	0.049	0.028	0.023	0.042	0.026	0.020	0.014	0.021	140
0.093	0.075	0.058	0.088	0.063	0.048	0.036	0.030	0.054	0.035	0.026	0.018	0.027	160
0.116	0.091	0.073	0.110	0.080	0.060	0.046	0.037	0.068	0.044	0.032	0.023	0.034	180
0.142	0.112	0.090	0.136	0.098	0.074	0.057	0.046	0.084	0.055	0.040	0.029	0.042	200
0.170	0.134	0.108	0.164	0.118	0.089	0.069	0.055	0.101	0.067	0.049	0.035	0.051	220
0.200	0.158	0.128	0.194	0.140	0.105	0.082	0.066	0.120	0.079	0.058	0.042	0.060	240
0.252	0.184	0.149	0.226	0.163	0.123	0.097	0.077	0.140	0.093	0.068	0.049	0.070	260
0.265	0.210	0.171	0.260	0.188	0.142	0.112	0.089	0.162	0.109	0.079	0.057	0.082	280
0.300	0.239	0.195	0.297	0.215	0.163	0.128	0.102	0.286	0.090	0.066	0.066	0.094	300
0.336	0.269	0.220	0.235	0.243	0.185	0.145	0.116	0.211	0.412	0.102	0.075	0.107	320
0.374	0.300	0.246	0.376	0.273	0.208	0.163	0.131	0.238	0.160	0.115	0.085	0.120	340
0.413	0.332	0.273	0.418	0.305	0.232	0.182	0.146	0.266	0.179	0.128	0.096	0.135	360
0.452	0.365	0.300	0.461	0.338	0.257	0.202	0.163	0.296	0.199	0.143	0.107	0.150	380
0.493	0.399	0.329	0.507	0.372	0.284	0.223	0.180	0.327	0.220	0.158	0.118	0.166	400
0.534	0.434	0.358	0.554	0.407	0.311	0.245	0.198	0.360	0.242	0.174	0.130	0.183	420
0.576	0.469	0.389	0.602	0.444	0.340	0.267	0.216	0.394	0.265	0.190	0.143	0.201	440
0.619	0.505	0.419	0.652	0.482	0.370	0.291	0.236	0.429	0.289	0.208	0.156	0.220	460
0.661	0.054	0.450	0.702	0.520	0.400	0.315	0.256	0.466	0.313	0.225	0.170	0.239	480
0.704	0.579	0.482	0.754	0.560	0.431	0.340	0.276	0.504	0.340	0.244	0.184	0.259	500

k_t

Beyond $b = 5$ kms we have for a zone with the width 1 km

$$k_t = 20.92 \frac{h^2}{b^2}$$

Actually, if we take the cogeoidal anomalies of the ordinary field (average density $1\ \text{station}/250\ \text{sq kms}$) *within the area I* as equal to the free air anomalies

$$g_{obs} + \triangle g_f - \gamma$$

$\triangle g_f$ being calculated from I: 4, the resulting error being in most cases less than 3 mgals. *And I am of the opinion that we are not far from the truth, if we consider these free air anomalies as measured cogeoidal ones with an accidental mean error of* ± 2 *mgals.*

Reductions of astrodeflections.

In order to compare gravimetrical results with astronomical determinations we have to reduce the measured latitudes and longitudes to cogeoidal level. With the aid of gravity measurements in the vicinity of astronomical stations, it is possible to correct the »measured» deflection components ξ_a and η_a for the difference of plumb line direction between the level surface of the station and the geoid, that is, to reduce them to »sea level». After this reduction has been made we may consider gravimetrical and astronomical *deflection differences* as equivalent within restricted areas (bibl. 13).

Rice (bibl. 16) uses the formulas for »sea level correction» deduced by Lambert (bibl. 11) for Stokes' point cogeoid:

$$\triangle \xi''_a = -0.''53\ h \sin 2\varphi - 0.''07 \frac{\partial h}{\partial x} h - 0.''65 \frac{\partial \triangle g}{\partial x} h$$

$$\triangle \eta''_a = \quad - 0.''07 \frac{\partial h}{\partial y} h - 0.''65 \frac{\partial \triangle g}{\partial y} h \qquad \text{I: 5}$$

The first term in $\triangle \xi_a$ is the correction for divergence towards the equator for the level surfaces of the theoretical earth. The others are, in order, the corrections due [1]) to the masstransport, [2]) to the divergence in the cogeoidal field. ξ is reckoned positive, when the level surface descends to the north (direction of x-axis), η being positive, when the level surface descends towards the east (direction of y-axis). The gradients are expressed in $^{m}/_{km}$ and $^{mgals}/_{km}$ respectively; h-factors are in *kms.* The anomaly gradients refer to the station, the elevation gradients are average values across the station.

We can, of course, use corresponding formulas for our reductions to Rudzki's cogeoid, if we are convinced, that they really take account of the very local curvature of the plumb line between station level and geoid. But before deciding to do so, certain points should be raised concerning the parts of reduction in I: 5. With regards the highest accuracy obtainable in determining gravity anomalies on the cogeoid (say, an accidental mean error of about + 1 mgal), it is useless to mention a real anomaly difference between two stations if this difference does not exceed at least 2 mgals. Now assuming the *error of representation* (E) for the cogeoidal anomalies in level land to be the same as *Hirvonens* (bibl. 9) we can extrapolate from his E-curve, and reach the conclusion that within areas of about 15 sqkms of plain country it will be difficult to detect any anomaly variations at all. Allowing for the greater error of representa-

tion (mean variation) within irregular terrain we may state that it generally would be useless to examine a more detailed cogeoidal field than the one constructed upon anomaly prismograms over areas of say 10 sqkms. Considering the mean anomalies over these unit areas we may describe such an accepted cogeoidal field by a parabolic function

$$\triangle g = a x^2 + b y^2 + 2 c x y + d x + e y + \triangle g_0$$

within areas of $6 \times 10 = 60$ sqkms, that is, a circle of 4.5 km radius. From 6 anomalies (e. g. at the centre and the corners of a symmetrical pentagon) we can determine the coefficients of this parabolic function, d and e being the anomaly gradients of the *accepted* cogeoidal field at the station. To obtain a rough evaluation of the mean error of these gradients, that is, of d and e, we have to express them in the six anomalies of the pentagon. It is easily seen that the mean error in d and e amounts to about $\pm$ 0.2 $^{\text{mgals}}/_{\text{km}}$ if the mean error of representation for each anomaly is $\pm$ 1 mgal. Even if we allow for twice that error of representation, the mean errors of calculated cogeoidal divergence should be only $\pm$ 0."1 for a height of 500 mts.

Of course, this minute error is not a correct measure of the accuracy in the computed divergence. For even, if we consider h to be exact, the error does not include the uncertainty due to the fact that we do not know the anomaly gradient of the *real* cogeoidal field at the station but only the gradient of the *accepted* field can be taken into account. Small undetectable anomaly differences may exist over short distances, which correspond to great local gradients, depending upon the *marked correlation between topography and cogeoidal anomaly*.

In I: 5 the term of the masstransport effect is the only one which takes the local topography directly into consideration. Rice, when taking account of »the average elevation gradient» across the station possibly means the slope of a smoothed-out topography (described as a 2nd degree function) over a circle of about 5 kms (the same area and method used in his investigations of mean anomaly gradients). In doing so, however, we may ignore adverse effects of steep topographical contours in the immediate neighbourhood of the station. The following example will show that this omission may give rise to considerable errors.

Assuming that the station is situated in the terrain as shown in fig. 1, we obtain, according to table II (see further below) the total effect of masstransport by constructing Stokes' point cogeoid (according to Rice), that is, the differential effect at P of topography and its condensation, equal to + 2."1.

From a consideration of a »2nd degree topography» within the 5 km circle we have the average gradient $\frac{\partial h}{\partial y}$ at the station equal to about 3.1 $^{\text{m}}/_{\text{km}}$, and thus the correction in η from the corresponding term in I: 5 amounts to + 0."1 (!) Naturally we get different results when we consider »average elevation gradients» over smaller areas. But which area should be chosen?

In order to take into account such influences of local topography, I consider it preferable to make a *finite* summation of the differential effect in horizontal attraction

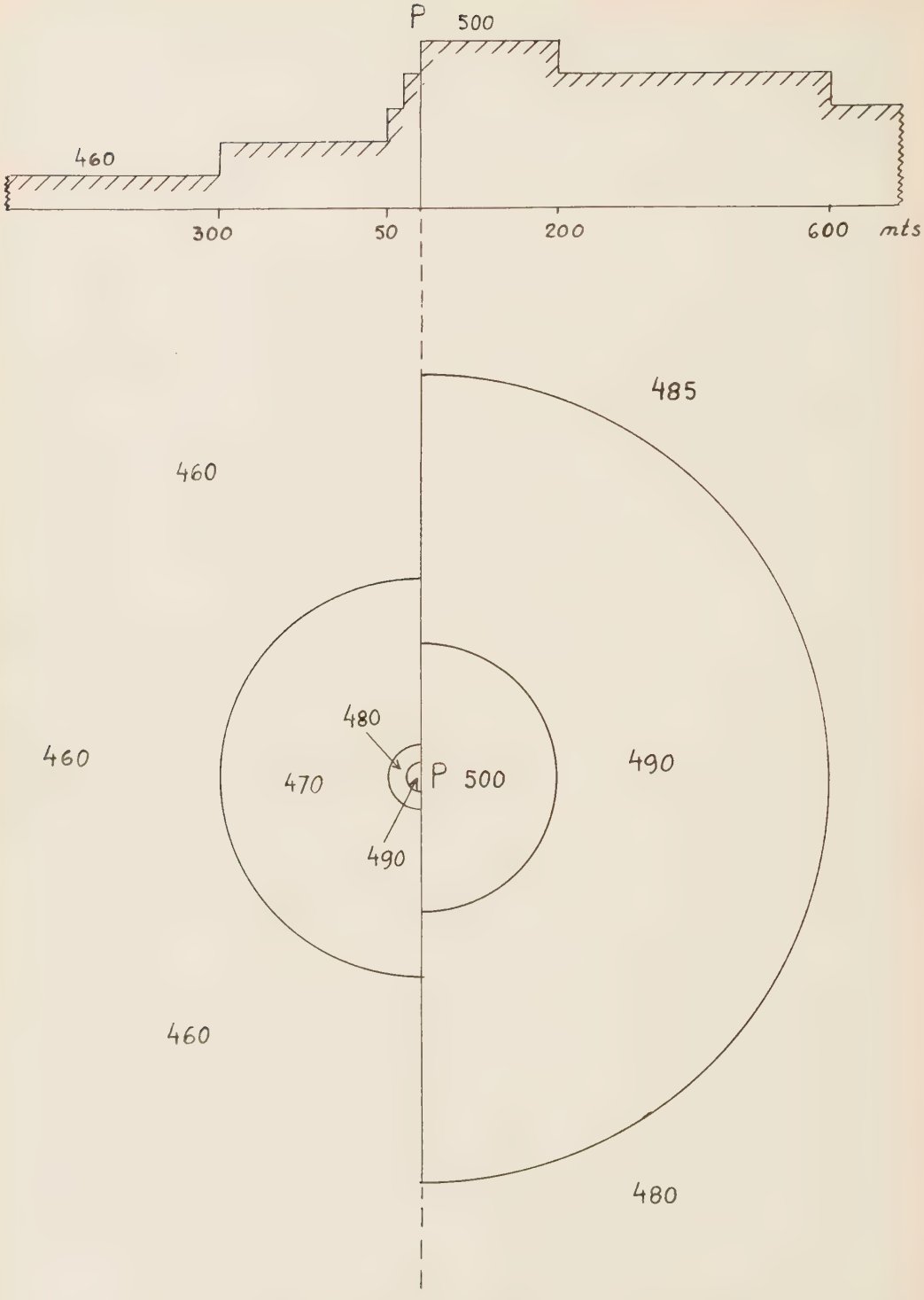


Fig. 1. All heights in mts. The outer heights are constant to infinity.

(*topography on P and »reflection» on* P_0), at the same time taking the divergence in the *Bouguer field* into consideration.

From an investigation of denser parts in the Bouguer field over the area A of ch II I conclude that the variation of Bouguer anomalies within small areas is considerably less than the variation of free air (or cogeoidal) anomalies, and this result coincides with Hirvonen's statement (bibl. 9 p. 7). Therefore, we may use the last terms in I: 5 to correct the differential effect of topography at P and its reflection at P_0 for the divergence in the Bouguer field, which is smoother and thus more accurate for interpolation than the free air (cogeoidal) field.

Denoting the effects of topography and its reflection by $-\triangle_t$ and $+\triangle_i$ respectively, the Bouguer divergence by $\triangle_B$ (for $\triangle\xi_a$, it should include the normal divergence according to the international formula of gravity), we have

$$\begin{aligned} \triangle\xi_a &= -\triangle\xi_t + \triangle\xi_B + \triangle\xi_i \\ \triangle\eta_a &= -\triangle\eta_t + \triangle\eta_B + \triangle\eta_i \end{aligned} \qquad \text{I: 6}$$

Here $\triangle_B$ can be taken from the expressions

$$\begin{aligned} \triangle\xi_B &= -0.''65\,\frac{\partial\triangle g}{\partial x}\,h - 0.''53\,h\sin 2\varphi \\ \triangle\eta_B &= -0.''65\,\frac{\partial\triangle g}{\partial y}\,h \end{aligned} \qquad \text{I: 6 a}$$

if the anomaly gradients are those of the Bouguer field. The order of steps in the deduced reduction is that indicated in I: 6.

Reckoning in the plane, we obtain for *the topographical effects from a compartment* with the radii b_1 and b_2 *kms*, the mean azimuth α and the mean height h *kms*, the wellknown expressions:

$$\begin{aligned} \delta\triangle\xi''_t &= -9.''94\cdot\varrho\cdot h\cdot\log\frac{b_2+\sqrt{b_2^2+h^2}}{b_1+\sqrt{b_1^2+h^2}}\cdot\cos\alpha \\ \delta\triangle\eta''_t &= -9.''94\cdot\varrho\cdot h\cdot\log\frac{b_2+\sqrt{b_2^2+h^2}}{b_1+\sqrt{b_1^2+h^2}}\cdot\sin\alpha \end{aligned} \qquad \text{I: 6 b}$$

if the station P is situated on the geoid ($h_p = 0$); ϱ is the mean density of the column of mass, referred to the compartment.

We may write the radial effect

$$\delta\triangle_t = -9.''94\,h\log\frac{b_2+\sqrt{b_2^2+h^2}}{b_1+\sqrt{b_1^2+h^2}}\cdot\varrho = -k''_d\cdot\varrho$$

and have

$$\begin{aligned} \delta\triangle\xi_t &= -k''_d\,\varrho\cos\alpha \\ \delta\triangle\eta_t &= -k''_d\,\varrho\sin\alpha \end{aligned} \qquad \text{I: 6 b}$$

The table II (pp.20,21) gives k''_d for different zones and mean heights and is calculated so densily that linear interpolation will usually suffice. To find k''_d for a zone compartment in relation to a station ($h_p = 0$), we have:

a) if $h_p > h$: take k''_d for h_p and subtract
k''_d for $h_p - h$

b) if $h_p < h$: take k''_d for h_p and add
k''_d for $h - h_p$

Beyond 4 kms, we have for a zone with $b_2 - b_1 = \triangle b = 0.5$ kms, the radial effect

$$\delta \triangle_t = - \frac{2.''16}{b} \cdot h \cdot \varrho = - k''_d \varrho$$

which is the effect on P_0. Here h is the mean height and b the mean radius of the zone compartment expressed in the same units. The formula is valid for $h_p = 0$, but with the same rules as above we obtain $\delta \triangle \xi_t$ and $\delta \triangle \eta_t$ even for other h_p : s.

The effect of the reflected topography at the projected station P_0 on the cogeoid can be taken from the same table. We have the radial effect $\delta \triangle_i = - k''_d \varrho$, where k''_d corresponds to the mean height of the zone compartment. Beyond $b = 4$ kms we have the radial effect

$$\delta \triangle_i = - \frac{2.''16}{b} \cdot h \cdot \varrho$$

if $\triangle b = 0.5$ kms.

The total radial effect from a zone compartment is then

$$\boxed{\delta(- \triangle_t + \triangle_i) = (\triangle k''_d \pm k''^{(\triangle h)}_d)\, \varrho} \quad \ldots\ldots\ldots\ldots \text{ I: 7}$$

Here $\triangle k''_d$ is the difference between k''_d for h_p and for h, $k''^{(\triangle h)}_d$ is the value of k''_d for $(h_p - h)$. The sign is used, when $h_p > h$, the $+$ sign, when $h_p < h$.

From an investigation of very extreme conditions for the terrain round P, we find that in the area A the formula I: 6 b and the table II will always suffice ($h_{max} = h_{p\,max} - 500$ mts). And I believe that scarcely any case exists, in which it is necessary to go farther from P than *2 kms*, if it is desired to calculate the topographical part of the reduction $(- \triangle_t + \triangle_i)$ with an error $< 0.''5$, assuming ϱ to be exact. In the example above, for instance, the omission of all terrain beyond 1 km affects $- \triangle \eta_t + \triangle \eta_i$ by less than $0.''1$. The total correction ($\varrho = 2.67$) is here $+ 2.''1$ (the same amount as we obtained for the masstransport: — topography + condenstion at P). The contributions from the different zones are:

Range		Contrib.	m_h
0—50	mts	$+ 0.''61$	$\pm$ 1 m
50—100	»	+ 0. 43	$\pm$ 2 »
100—200	»	+ 0. 45	$\pm$ 5 »
200—500	»	+ 0. 43	$\pm$ 5 »
500—1.000	»	+ 0. 16	$\pm$ 5 »
1.000—	∞	< 0. 1	

Determining mean heights in zone compartments with the constant aperture of 20^c and assuming the error of each mean height to be random error, we can make a rough estimate of the total mean error in $-\triangle_t + \triangle_i$. Taking the mean error of one mean height equal to m_h above, we obtain in our example the total mean error $\sim \pm 0.''1$. Using good topographical (and geological) maps in the vicinity of the astronomical station, I believe it will always be possible to determine this part of the reduction within $0.''3$ (allowing for the inaccuracy of the chosen ϱ-value).

The additional correction for divergence in the Bouguer field can be calculated from the expressions I: 6 a (p. 17), the gradients being computed from the Bouguer anomalies in the vicinity of the station. These anomalies should be determined with the highest accuracy obtainable, that is, with respect to local curvatures and with the use of table I. Accepting a Bouguer field, which is constructed upon mean anomaly prismograms of the same size as proposed for the free air anomalies on p. 15, we may hope to obtain the gradients with an error which does not appreciably exceed the mean error of d and e deduced at the same place. Of course, a greater density than $^{1\ \text{station}}/_{10\ \text{sq kms}}$ will reduce the mean error of the gradients in this *accepted* Bouguer field, but the additional work involved appears to me unnecessary.

Using the method for calculation of »sea level corrections», discussed on the preceeding pages, I believe that it will usually be possible to obtain this reduction with an error which does not exceed $0.''5$.

Table II

(Rules for use, see p 18)

Zone h mts	0 -25 mts	25—50	50—75	75— 100	100— 150	150— 200	200 - 250	250— 300	300 - 350	350— 400	400 500	500— 600	600 700
20	0.0900	0.0520	0.0333	0.0242	0.0346	0.0247	0.0188	0.0157	0.0133	0.0115	0.0191	0.0157	0.013
40	0.1017	0.0790	0.0585	0.0451	0.0666	0.0484	0.0380	0.0312	0.0264	0.0229	0.0384	0.0314	0.026
60	0.1049	0.0914	0.0749	0.0612	0.0943	0.0704	0.0558	0.0461	0.0393	0.0342	0.0573	0.0470	0.039
80	0.1062	0.0975	0.0851	0.0729	0.1171	0.0902	0.0726	0.0604	0.0516	0.0451	0.0759	0.0623	0.052
100	0.1068	0.1009	0.0915	0.0812	0.1354	0.1075	0.0879	0.0739	0.0636	0.0557	0.0940	0.0774	0.065
120	0.1070	0.1028	0.0957	0.0872	0.1498	0.1224	0.1018	0.0865	0.0749	0.0659	0.1117	0.0923	0.078
140	0.1072	0.1041	0.0985	0.0915	0.1612	0.1351	0.1142	0.0981	0.0855	0.0756	0.1287	0.1068	0.091
160	0.1074	0.1050	0.1005	0.0946	0.1701	0.1458	0.1253	0.1087	0.0954	0.0848	0.1451	0.1209	0.103
180	0.1075	0.1056	0.1019	0.0970	0.1772	0.1549	0.1350	0.1183	0.1047	0.0935	0.1608	0.1346	0.115
200	0.1076	0.1060	0.1030	0.0988	0.1829	0.1625	0.1435	0.1271	0.1132	0.1016	0.1758	0.1479	0.127
220	0.1077	0.1063	0.1038	0.1002	0.1877	0.1690	0.1510	0.1349	0.1211	0.1093	0.1901	0.1607	0.138
240	0.1077	0.1065	0.1044	0.1014	0.1914	0.1744	0.1575	0.1420	0.1283	0.1164	0.2036	0.1730	0.149
260	0.1077	0.1067	0.1049	0.1022	0.1945	0.1791	0.1632	0.1483	0.1349	0.1230	0.2164	0.1848	0.160
280	0.1077	0.1069	0.1053	0.1030	0.1971	0.1830	0.1683	0.1540	0.1409	0.1292	0.2285	0.1962	0.171
300	0.1078	0.1070	0.1056	0.1036	0.1992	0.1864	0.1727	0.1591	0.1464	0.1349	0.2399	0.2070	0.181
320	0.1078	0.1071	0.1058	0.1040	0.2010	0.1894	0.1766	0.1637	0.1515	0.1402	0.2506	0.2174	0.190
340	0.1078	0.1072	0.1060	0.1044	0.2026	0.1919	0.1800	0.1678	0.1561	0.1450	0.2606	0.2273	0.200
360	0.1078	0.1073	0.1062	0.1048	0.2039	0.1941	0.1830	0.1715	0.1602	0.1495	0.2700	0.2367	0.209
380	0.1078	0.1074	0.1064	0.1052	0.2050	0.1960	0.1857	0.1748	0.1640	0.1536	0.2788	0.2456	0.218
400	0.1078	0.1074	0.1066	0.1054	0.2060	0.1977	0.1881	0.1778	0.1677	0.1575	0.2871	0.2541	0.226
420	0.1078	0.1075	0.1067	0.1056	0.2069	0.1992	0.1902	0.1806	0.1707	0.1610	0.2948	0.2622	0.234
440	0.1078	0.1075	0.1067	0.1058	0.2076	0.2005	0.1922	0.1830	0.1736	0.1643	0.3020	0.2699	0.242
460	0.1078	0.1075	0.1069	0.1060	0.2083	0.2017	0.1939	0.1852	0.1763	0.1673	0.3087	0.2771	0.249
480	0.1078	0.1076	0.1070	0.1061	0.2088	0.2028	0.1954	0.1873	0.1787	0.1701	0.3151	0.2840	0.256
500	0.1079	0.1076	0.1071	0.1063	0.2094	0.2037	0.1968	0.1891	0.1810	0.1726	0.3210	0.2905	0.263

k^{w}_{d}

700—600	800—900	900—1000	1000—1200	1200—1400	1400—1500	1600—1800	1800—2000	2000—2500	2500—3000	3000—3500	3500—4000	zone h mts
0.0115	0.0102	0.0091	0.0157	0.0133	0.0115	0.0102	0.0091	0.0193	0.0158	0.0133	0.0115	20
0.0230	0.0203	0.0182	0.0315	0.0266	0.0231	0.0203	0.0182	0.0386	0.0315	0.0266	0.0231	40
0.0345	0.0304	0.0272	0.0472	0.0399	0.0346	0.0305	0.0273	0.0578	0.0472	0.0399	0.0346	60
0.0459	0.0405	0.0363	0.0628	0.0532	0.0460	0.0406	0.0364	0.0771	0.0630	0.0532	0.0461	80
0.0572	0.0505	0.0452	0.0784	0.0664	0.0575	0.0508	0.0454	0.0963	0.0787	0.0665	0.0576	100
0.0683	0.0604	0.0542	0.0939	0.0795	0.0690	0.0609	0.0545	0.1155	0.0944	0.0798	0.0692	120
0.0793	0.0702	0.0630	0.1093	0.0926	0.0804	0.0710	0.0635	0.1347	0.1101	0.0931	0.0807	140
0.0902	0.0800	0.0718	0.1248	0.1057	0.0917	0.0810	0.0725	0.1538	0.1258	0.1064	0.0922	160
0.1009	0.0895	0.0804	0.1399	0.1187	0.1030	0.0910	0.0815	0.1729	0.1414	0.1196	0.1037	180
0.1114	0.0990	0.0890	0.1549	0.1316	0.1143	0.1010	0.0905	0.1920	0.1571	0.1329	0.1151	200
0.1216	0.1083	0.0975	0.1698	0.1444	0.1255	0.1109	0.0994	0.2110	0.1727	0.1461	0.1266	220
0.1317	0.1174	0.1058	0.1846	0.1571	0.1366	0.1208	0.1083	0.2300	0.1883	0.1593	0.1381	240
0.1416	0.1264	0.1140	0.1992	0.1697	0.1477	0.1307	0.1172	0.2489	0.2038	0.1725	0.1495	260
0.1512	0.1352	0.1221	0.2136	0.1822	0.1587	0.1405	0.1260	0.2678	0.2193	0.1857	0.1610	280
0.1605	0.1438	0.1301	0.2278	0.1945	0.1696	0.1502	0.1348	0.2866	0.2348	0.1988	0.1724	300
0.1696	0.1522	0.1379	0.2418	0.2068	0.1804	0.1599	0.1435	0.3053	0.2502	0.2120	0.1838	320
0.1784	0.1604	0.1456	0.2556	0.2189	0.1912	0.1695	0.1522	0.3240	0.2657	0.2251	0.1952	340
0.1870	0.1685	0.1531	0.2692	0.2308	0.2018	0.1791	0.1609	0.3426	0.2810	0.2382	0.2066	360
0.1953	0.1763	0.1604	0.2825	0.2427	0.2123	0.1885	0.1695	0.3611	0.2963	0.2512	0.2180	380
0.2033	0.1839	0.1676	0.2957	0.2543	0.2228	0.1980	0.1780	0.3795	0.3116	0.2642	0.2293	400
0.2111	0.1914	0.1746	0.3086	0.2659	0.2331	0.2073	0.1865	0.3978	0.3268	0.2772	0.2406	420
0.2186	0.1986	0.1815	0.3213	0.2772	0.2434	0.2166	0.1950	0.4161	0.3420	0.2902	0.2519	440
0.2258	0.2056	0.1882	0.3338	0.2885	0.2535	0.2258	0.2033	0.4342	0.3572	0.3031	0.2632	460
0.2328	0.2124	0.1948	0.3460	0.2995	0.2635	0.2348	0.2116	0.4523	0.3722	0.3160	0.2745	480
0.2396	0.2190	0.2012	0.3580	0.3104	0.2734	0.2439	0.2199	0.4702	0.3872	0.3289	0.2857	500

k''_d

Beyond b = 4 kms we have for a zone compartment of width 0.5 km

$$k''_d \quad \frac{2.16\ h}{b}$$

II.

The Swedish gravity field and its surroundings.

In order to obtain a gravity field on the cogeoid for integration with Stokes' formula, I have used gravity data from the whole earth. In most cases the cogeoidal anomalies are sufficiently close to the free air anomalies. Both these anomalies show a very marked correlation with topography. Thus a mountain station usually has a greater (algebraically) anomaly than neighbouring valley stations. And the anomalies at sea depend greatly upon the depth at the station. Greater depths give algebraically smaller anomalies than more shallow waters.

If we integrate directly upon an anomaly field that has been constructed through linear interpolation between the given anomalies without taking account of the correlation mentioned above, the mean anomalies of the integration compartments may be more or less *systematically* incorrect. The error is proportional to the difference $h_m - h_i$ between the mean height and the interpolated average height of stations in the compartment. I have tried to eliminate this error by considering a linear regression between cogeoidal anomaly f and topographical height (ocean depth) h (see Jeffreys, bibl. 10)

$$f = a + b\,h \qquad \text{II: 1}$$

a and b being treated as independent of h. We may assume that in irregular terrain (profile of depths) a will vary more regularly than f, because it mainly depends upon density anomalies in the rigid crust below the geoid, and their variation does not usually show such a pronounced undulations as the irregular topography. b is as a rule positive for land stations, and negative for sea stations. If the density in the crust were ~ 2.7, we should obtain b for land $\sim +100$ $^{\text{mgals}}/_{\text{km}}$, for sea ~ -60 $^{\text{mgals}}/_{\text{km}}$. Calculating b from neighbouring stations of different heights (depths) within an area that is not too small and taking the average b_m for that area we usually find b_m to be less than the normal values above. This indicates a tendency towards compensation of massdifferences between such areas. Thus Jeffreys (l. c.) finds in his investigation of 10° degree squares the following values of b in these squares for the whole earth

$$b_{\text{land}} = +46 \pm 30\ ^{\text{mgals}}/_{\text{km}}$$

$$b_{\text{sea}} = -22 \pm 14\ ^{\text{mgals}}/_{\text{km}}$$

arithmetical means of upper and lower sextiles, 30 and 14 being the semiintersextile ranges.

When integrating for absolute deflection components at Östra Älvhöjden (ch III, VII) I have used three different parts of the earth's gravitational field, namely

I: A circular area with the radius 190 kms,

II: The area between 190 kms and 990 kms,

III: The area from 990 kms to the antipode.

In II and III the smallest integration compartments subtend areas of ~ 4.300 sqkms and ~ 97.000 sqkms respectively, the former being about 50 p.c. of an average 1° square, the latter about 11 p.c. of an average 10° square. For the calculation of curvatures within the central part of area I (ch IV, VI) the gravity area I is used out to a radius of 100 kms, the area II from 100 kms to 990 kms and the area III from there to the antipode. In accordance with the aim of the investigations in this paper, I have thus constructed three fields of cogeoidal gravity:

A: *The Wideland gravity field* within a circle round Östra Älvhöjden with the radius ~ 200 kms, including all measurements of gravity made by Swedish and Norwegian observers (average density about 1 $^{\text{station}}/_{250\ \text{sq kms}}$).

B: *The European field* within a circle round the same point of 1.500 kms radius (maximum density used 1 $^{\text{station}}/_{2.000\ \text{sq kms}}$). The data are collected from several gravity catalogues (see further below).

C: *Globular field* of the whole earth constructed upon Jeffreys' (l. c.) free air anomalies of 10° squares.

The fields A, B and C have been illustrated in the following manner.

The A — field.

If the stations (with the density 1/250) were evenly distributed and no correlation with height existed, the free air anomaly at each station would represent the mean free air anomaly within a square of 16 × 16 sqkms, the station being at the centre and the mean error of representation (mean variation) within the square not exceeding 5 mgals (see Hirvonen bibl. 9). To obtain a continous field the only possible course consists in smoothing out the anomalies to a surface. In practice this construction consists in drawing isanomaly curves through nearly linear interpolation between the centres of the prismogram squares. This means: within areas of at least 10 × 10 sqkms we may consider the surface as a plane and within areas of about 25 × 25 sqkms we consequently describe it by a parabolic function (six points determine a surface of 2nd degree). We could accept this method for constructing the details of the A field if b were known to be 0. An investigation of the area A shows, however, that b usually assumes positive values, which vary from one part of the area to another. The values of b, inserted for every 1° long × 0.°5 lat-square in the b_m-field of the area A (fig. 2), have been calculated as least-square solutions for all stations within every square, each station giving an observation equation of the form $f_\nu = a + b \cdot h_\nu$, and the resulting values of b obviously represent the means for the stations. The mean errors have also been calculated, and the result b_m for the coefficient of regression has been referred to the »gravity» centre of the stations within the square.

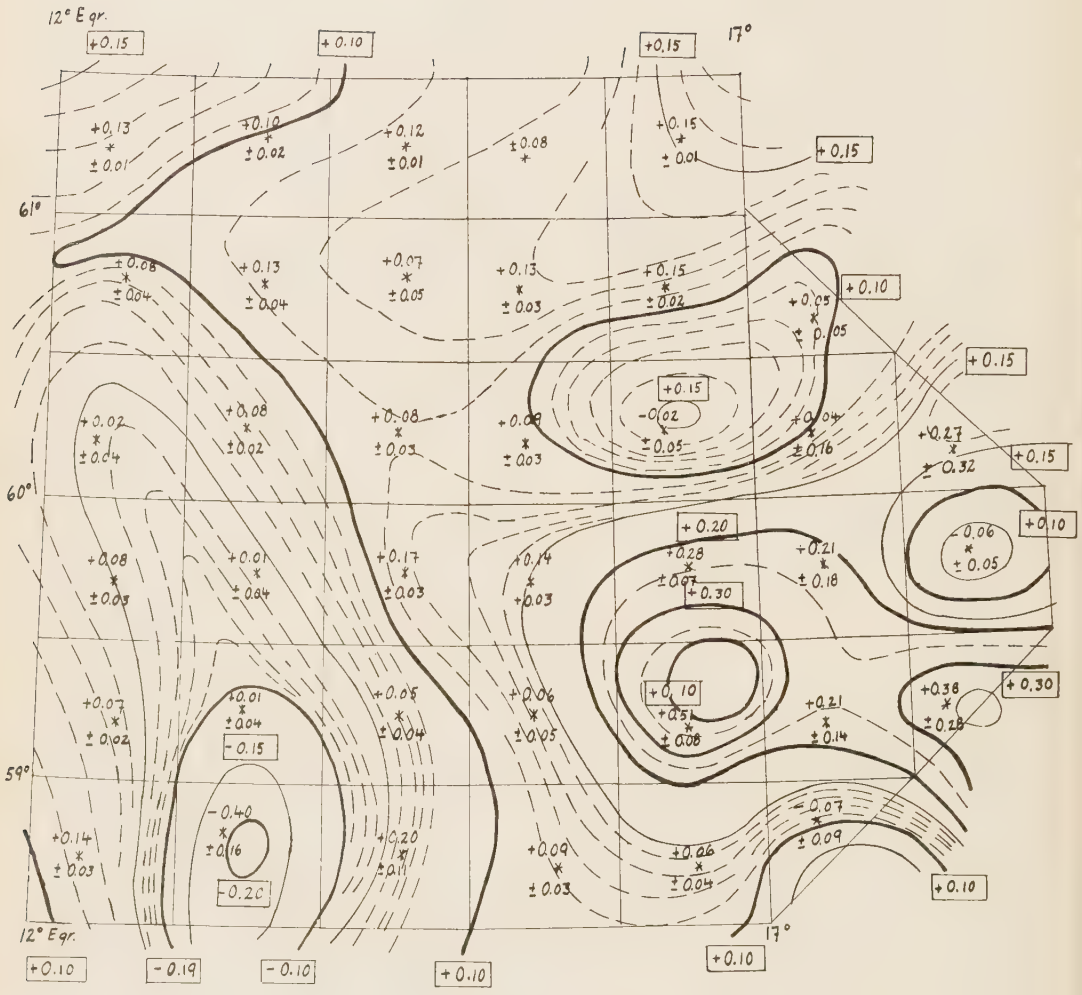


Fig. 2.

The bm-field of area A.

The curves of the b_m-field have then been drawn through interpolation between the b -values, account being taken of the various mean errors. The median of the mean errors in calculated b -values, representing squares of the magnitude $\sim$ 3.000 sqkms is about $\pm$ 0.04 $^{mgals}/_{m}$, the actual mean error being set beside each b_m-value. The median of all b_m-values is about $+$ 0.08 $^{mgals}/_{m}$.

In order to investigate what degree of accuracy can be obtained by using this b_m-field to interpolate a denser anomaly field from the given free air anomalies, I have proceeded as follows:

The density $^{1\ p.}/_{250\ sqkms}$, which has been considered as an average over the area A, corresponds to about 12 stations within a square of 3.000 sqkms. For 12—13 stations at the most, evenly distributed within each square, I have calculated the quantities $a_\nu = f_\nu - b_m \cdot h_\nu$. From these a-values an a-field has been constructed through approximately linear interpolation. The b_m-values have been taken as point values from the b_m-field described above. From those gravity stations, which have not been used in the construction of the a-field, we may now check the validity of our interpolation method. Calculating $f = a + b_m \cdot h$ for these stations, we obtain anomalies, which will differ more or less from the *known* anomalies. In the figure 3 the differences between the measured and the interpolated values are set beside each station. It is obvious that in the densist parts of the anomaly field there is a certain tendency for runs of the signs in these differences. Assuming that the *accidental variation* of the differences *begins* when we compare tightly adjacent squares of the magnitude 50—100 sqkms (and this may possibly be stated from the fig. 3), we should be able to regard mean free air anomalies for such areas as stochastically independent.

In order to testify, whether we may use Gauss' law for propagation of errors when calculating mean anomalies over larger squares, I have studied the distribution of the aforesaid differences, taking means within such small squares over the densist regions, and using single values within more sparsely filled areas. In fig. 4 the distribution of the differences shows no typical departure from the normal law of distribution. *The standard error* calculated from this material is about $\pm$ 2.7 mgals; (the standard error without any sampling is $+$ 2.9 mgals). *In the following, I have accepted mean free air anomalies $f_m = a_m + b_m \cdot h_m$ for squares of 50 sqkms within the area A as »observed» mean free air anomalies having an accidental mean error of $\pm$ 2.7 mgals, as far as h is considered to be exact.* a_m should be taken from the a-field, b_m from the b_m-field and h_m from a map of heights. For more extended lakes with greater depths (parts of the lake of Vänern and the lake of Vättern) I have also used the b_m-field taking h_m equal to the height of the surface, minus 2/3 of the mean depth, when the latter exceeds 10 mts. In this way approximate account is taken of the smaller attraction of water (density $\sim$ 1/3 of the topography). For those parts of the Baltic, which are included in the area A, Jeffreys' world mean (median) of b_{sea} ($-$ 0.02 $^{mgals}/_{m}$) has been used.

Assuming that correct mean free air anomalies for squares of 50 sqkms represent (see the investigation of abnormal curvatures in ch I) the cogeoidal field with the same accuracy, set out for point values in ch I and taking account of the mean error in h

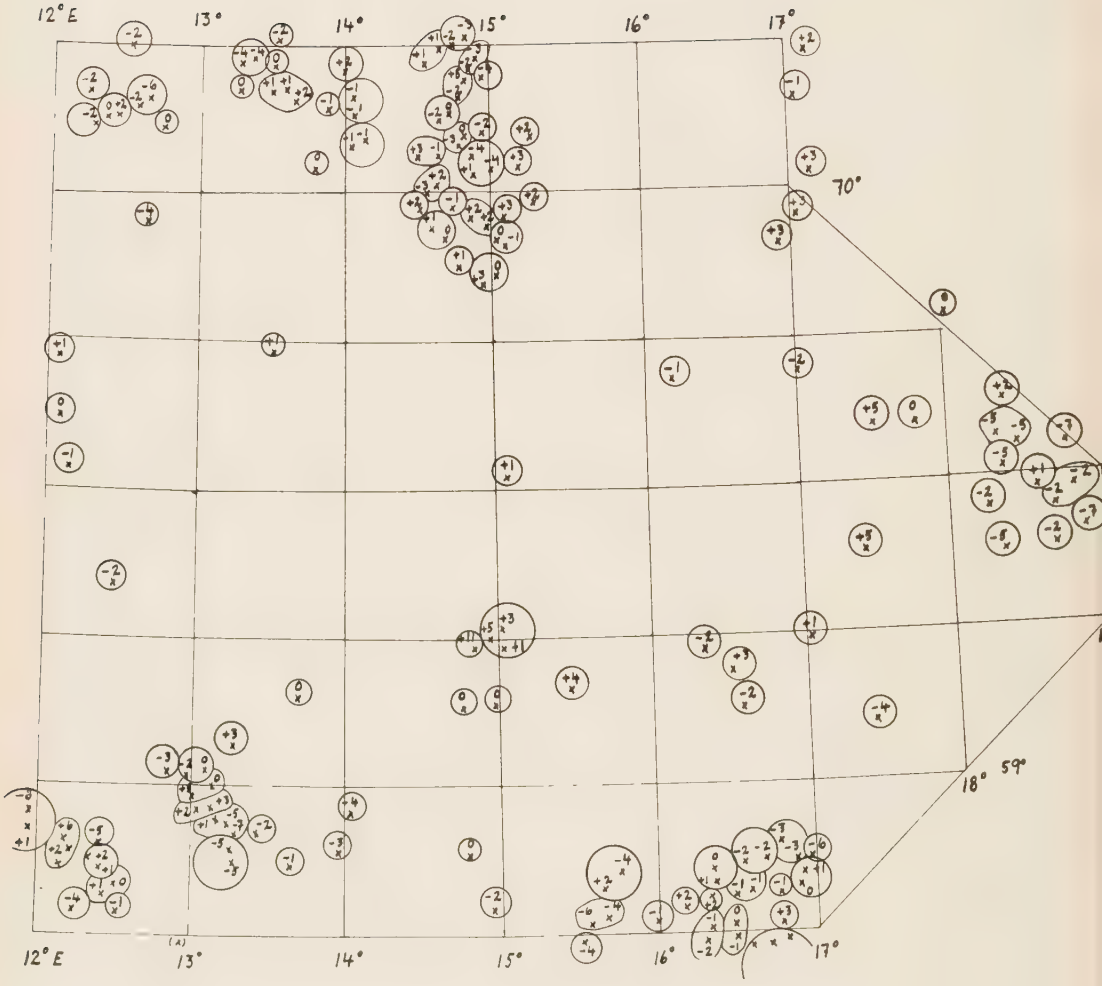


Fig. 3.

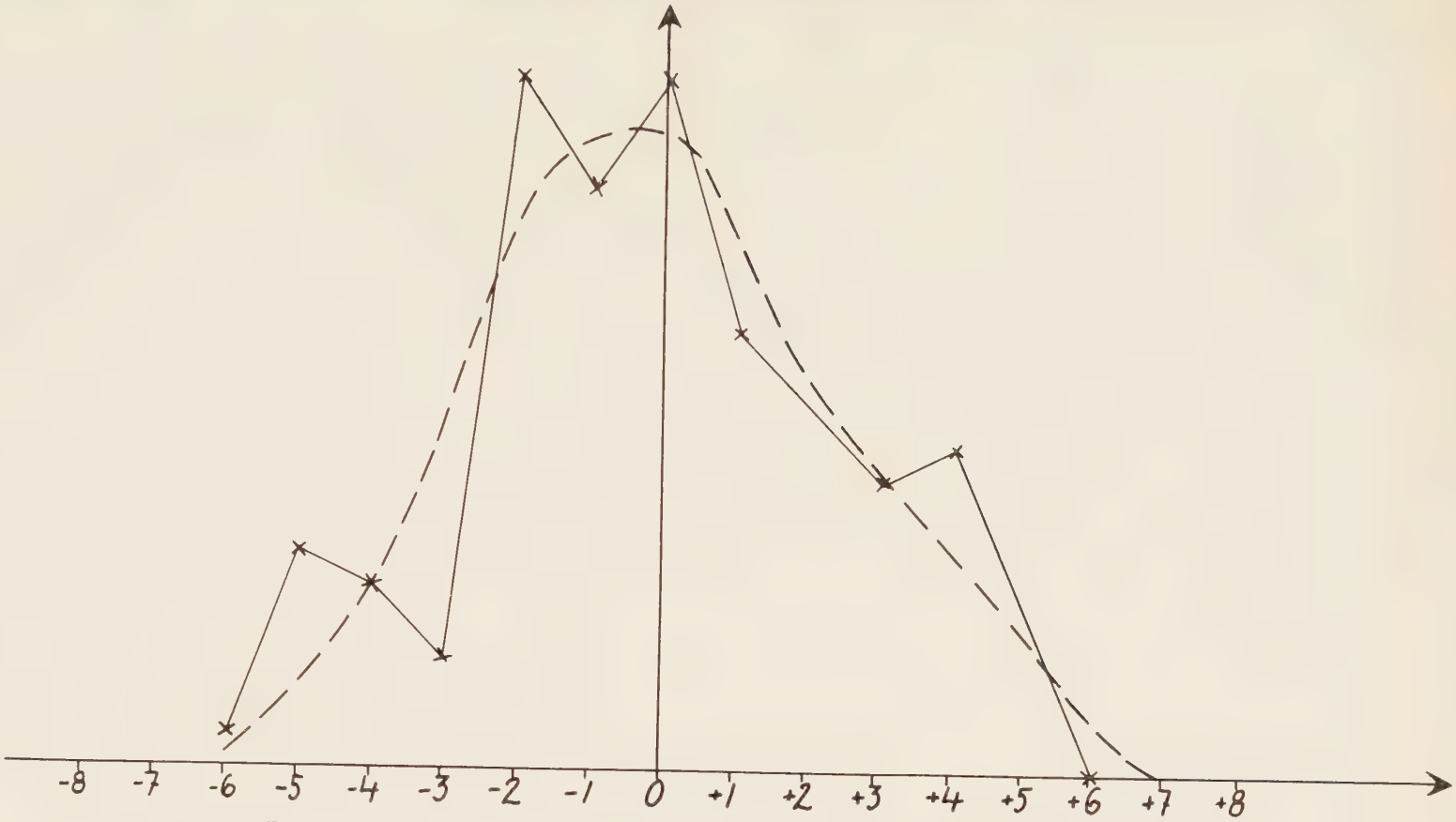


Fig. 4. Horizontal: measured—interpolated value in mgals. Vertical: Number of differences.

evaluated for the same squares on the maps of heights (bibl. 2), we obtain a total accidental mean error for areas of 50 sqkms

$$M_o = \pm \sqrt{2.7^2 + 2.0^2 + b_m^2 \cdot M^2 h_m}$$

If we use the median value + 0.08 for b_m within the area A and take $M_h = \pm 15$ mts (see bibl. 20 p. 128), we *obtain as an average*

$$\boxed{M_o = \pm 3.6 \text{ mgals}}$$

for squares of the size 7 × 7 sqkms.

An f_m-field has now been computed as mean anomalies of such squares (map I). In using this f_m-field, we obtain the following mean errors (m) of mean cogeoidal anomalies for *integration compartments* (T), applied by calculating a) deflections, b) curvatures (chs III, IV, VI and VII):

a) *Deflections.*

Zone	T sqkms	$\sqrt{0.02\ T}$	m	m′	n	m_E
			±	±		±
6	72	1.2	3.0	6.0	0.3	4.3
7	124	1.6	2.3	4.6	0.5	4.6
8	258	2.3	1.6	3.2	1.0	4.6
9	438	3.0	1.2	2.4	1.8	4.4
10	797	4.0	0.9	1.8	3.2	3.9
11	1480	5.4	0.7	1.4	5.9	3.4
12	2630	7.2	0.5	1.0	10.5	2.9

b) *Curvatures.*

Zone	T sqkms	$\sqrt{0.02\ T}$	m	m′	n	m_E
			±	±		±
12	453	3.0	1.2	2.4	1.8	4.3
13 a	296	2.4	1.5	3.0	1.2	4.6
13 b	632	3.5	1.0	2.0	2.5	4.2

Here n is the number of measured anomalies within the compartment, according to the accepted average density $^{1\ p.}/_{250\ sqkms.}$ m_E is the mean error of representation for level land, if we omit any interpolation. It is calculated from the result of Hirvonen's (bibl. 9) investigation of Swedish free air anomalies and account is taken of the accepted uncertainty in cogoidal representation within the unit area 50 sqkms (± 2 mgals). m' in the fifth column of the tables is the mean error of a mean cogeoidal anomaly taken from a continuous free air field, which has been constructed by nearly linear interpolation between Wideland's point values. The standard error calculated for the same samples as used above in checking our special interpolation method amounts to about ± 7.2 mgals. m' also includes the uncertainty of cogeoidal representation (similarly to m and m_E). In order to demonstrate also the greater smoothness within restricted areas of the a-field, compared to corresponding areas of the original field

of Wideland, I have calculated the standard deviations for all station values of the two fields within 1° long × 0,°5 lat — squares with irregular terrain conditions (22 squares with standard deviations of mean heights for unit areas (50 sqkms) excceding ± 50 mts). For the *a*-field I obtain the median standard deviation for such a square $\delta_a = \pm 6$ mgals and for the Wideland field $\delta_w = \pm 9$ mgals.

Our method thus seems to be advantageous as regards the accidental errors, but, above all, we should bear in mind, that at the same time, the mean cogeoidal anomalies will be more or less freed from the systematic error — $b_m (h_m - h_i)$.

For the *calculation of deflections* (reductions of ch V and ch VI, absolute deflection ch VII) we have proposed a greater density of the gravity stations within the inner radius of deflection zone 6. I have been informed, that dense measurements really exist in the vicinity of some of my deflection points (ch V), but as the results are not yet published, I have endeavoured to *demonstrate* all computations within zone 6 by using the b_m-values down to squares of the size 3 × 3 sqkms (chs VI, VII).

By *investigating the geoidal structure* over the central part of the area A, I have used a smoothed-out cogeoidal field, which corresponds to a somewhat generalized geoid, defined in the following manner: We consider mean anomalies over squares of about 530 sqkms and describe a smooth field by constructing a continuous surface over the prismogram of these mean anomalies, taken as point values at the »gravity» centres of the stations within the squares. In this smoothed—out field we may ordinarily regard the anomaly surface as a plane over areas of about 16 × 16 sqkms and as a surface of 2nd degree,

$$f = Ax^2 + By^2 + 2\,Cxy + Dx + Ey + f_o,$$

over areas of about 3.200 sqkms (six »points» determine the coefficients in the equation of the surface). The area 3.200 sqkms is equal to that included within the inner radius of curvature zone 12. Thus integrating in the f -field for the curvature zone 12 and two parts (13 a + 13 b) of zone 13 only, we must use the parabolic representation over a circle of the radius 32 kms.

The field described above, together with the *B*- and *C*-fields below define the generalized geoid in question.

All anomalies used in the construction of the field over the area A refer to the Potsdam standard with the following *g*-values accepted for the Swedish and Norwegian reference stations.

Stockholm (»static« point)..... 981 846.7 mgals
Oslo..... 981 928.4 mgals

The B-field.

Here, too, I have endeavoured to use the *a*-values of the linear regression $f = a + bh$ for interpolation, also intending to reduce the systematic effect $b_m (h_m - h_i)$. As there are extended land areas here with very few gravity stations and as there is an almost complete lack of observations for open sea regions, I have not yet considered it necessary to carry out any investigations of b in all *known* details of the field. For landsquares

of the area B, a b -value differing slightly from Jeffreys' world median (see p. 22) has been used. With regards 10°—landsquares his value ($+ 0.05$ $^{mgals}/_{m}$) seems to characterize widely spread regions within the area fairly satisfactorily. In fig. 5 we note the results of b_m for 10°—squares within the area B, obtained by Jeffreys, and for some smaller squares obtained by him (*III a—III d*), by Heiskanen (*I a*; bibl. 7) and by myself (*I b*). The median of b for the investigated 10°—squares is about $+ 0.06$ $^{mgals}/_{m}$, and the median of the calculated mean errors (obtained from least square solutions) is approximately ± 0.02 $^{mgals}/_{m}$. The same result is obtained, if we compute the weighted mean of the 10°-squares and its mean error with respect to the individual mean errors. For the weighted mean we obtain $+ 0.056$ $^{mgals}/_{m} \pm 0.008$ $^{mgals}/_{m}$ (mean of six squares). The value $+ 0.06$ $^{mgals}/_{m}$ will then represent one average 10°-square (area $\sim$ 750.000 sqkms) with a mean error of about ± 0.02 $^{mgals}/_{m}$. Taking b_m as a constant equal to $+ 0.06$ $^{mgals}/_{m}$ it will probably represent an average 5°-square (area $\sim$ 190.000 sqkms) with a standard error of about ± 0.03 $^{mgals}/_{m}$, this number being obtained from the differences between the constant value $+ 0.06$ and the values of the squares *I a*, *I b*, *III a—III d*.

For smaller land areas we have a direct means of deciding the degree of accuracy which can be obtained in mean free air anomalies by using the formula $f_m = a_m + + 0.06\ h_m$ for ordinary terrain conditions, maximum station density and great measuring accuracy within the B-field. The square *I b* is identical with the area A. I have calculated mean free air anomalies over squares of about 3.000 sqkms within this area, obtained from the f_m-field and having accidental mean errors of the magnitude ± 0.4 mgals (see above). Thus by comparing the corresponding anomalies of the B-field to those of the A-field, we may clearly neglect the mean errors of the latter. See further below.

The B-field has been built up from point values of free air anomalies as follows: The density aimed at has been $^{1\ p.}/_{1.200\ sqkms}$, which corresponds to about 5 stations within an 1°-square (1° long × 1° lat) at the latitude 60°. Naturally, it has only been possible to realize this for landsquares and for these only infrequently. Each gravity station on land gives a value of a equal to $f - 0.06\ h$, f being the free air anomaly according to the international formula for gravity and referred to the Potsdam standard; the accepted g-values for the European reference stations are taken from the list of Tanni (bibl. 19 pp 47, 48). The total number of land stations is about 1.600. The free air anomalies are collected from the following sources:

Report of Borrass	1909	271	stations	See bibl. 19 pp 50—53
» » »	1912	12	»	
» » Soler	1922	39	»	
» » »	1924	1	»	
» » »	1927	9	»	
» » »	1930	48	»	
» » »	1933	86	»	
» » »	1936	111	»	
» » »	1939	136	»	

Fig. 5. b-values in the B-field.

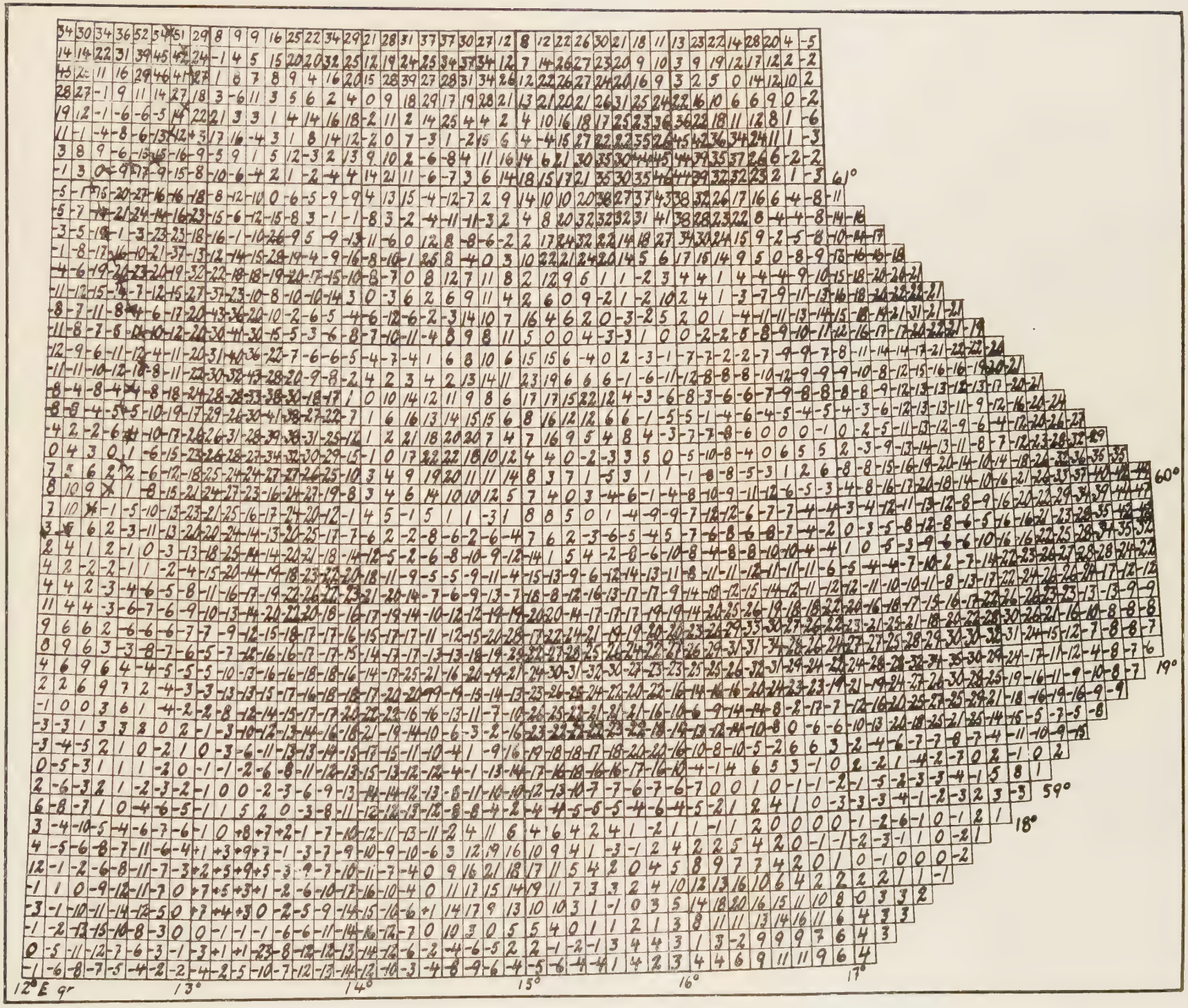
Report of Haasemann	1916	23	stations	
» » Berroth and Schmehl	1931	16	»	
Catalogue of Kasansky	1933	198	»	See
» » Schmehl	1933	2	»	bibl.
» » Oczapowski	1933	22	»	19 pp
» » Haalck (land stations)	1933—34	8	»	50—53
Catalogue of Hirvonen	1937	102	»	
» » Heiskanen	1939	92	»	
Catalogues » Wideland	1946, 1951	355	»	(bibl. 21, 22)
A manuscript list from Norges Geografiske Opmåling	1953	60	»	

As regards sea stations, the material available within the area *B* is too sparse to provide satisfactory information of mean anomalies over more extended sea regions. In most cases, they are situated within restricted areas not far from continental or island coasts. In the construction of the *B*-field 33 sea stations have been used. They are taken from the rather inaccurate measurements of Haalck in 1933 (2 stations near the island of *Helgoland*, 4 stations near the southern and eastern coast of *Skåne*, 6 stations in the *Gulf of Finland*), of Norgaard in 1935 (19 stations in the *Skagerack* and *Kattegatt*), and of Vening Meinez 1934—39 (1 station in the *Channel*, 1 station at the *North Sea*).

For each sea station an *a*-value equal to $f + 0.02\ d$, has been calculated, *d* being the depth at the station in *mts*. The b_{sea}-value -0.02 $^{mgals}/_{m}$ is the world median for 10°-squares according to Jeffreys. It has been chosen because it nearly coincides with the weighted mean of b_{sea} for such 10°-squares which are situated immediately outside the *B*-field (bibl. 10 p. 18—19). For some smaller areas we have a means of checking the b_{sea}-value chosen. Norgaards measurements, mentioned above, provide three squares of about 3.000 sqkms, each containing stations with marked differences of depth and giving a least square solution for a number of equations of the form $f = a + b \cdot d$. The three b_{sea}-values obtained are -0.05 $^{mgals}/_{m} \pm 0.02$ $^{mgals}/_{m}$, 0.00 $^{mgals}/_{m} \pm 0.01$ $^{mgals}/_{m}$ and -0.06 $^{mgals}/_{m} \pm 0.08$ $^{mgals}/_{m}$, the weighted mean being -0.01 $^{mgals}/_{m} \pm 0.01$ $^{mgals}/_{m}$, which is not inconsistent with Jeffreys' world median -0.02 $^{mgals}/_{m} \pm 0.01$ $^{mgals}/_{m}$ (see p. 22).

Most of the land stations in the *B*-area, which have been used for the construction of the cogeoidal field, are pendulum stations. In Sweden, Norway and Finland alone have gravimetrical measurements been taken into account (for *Sweden* all 355 stations are gravimetrically determined). In *Norway* the 60 stations from the above-mentioned manuscript list are gravimetrical points (the number of old pendulum stations being 46). In the Northern part of *Finland* account has been taken of isanomaly curves plotted from gravimetrical determinations on the chart of Pesonen 1951 (bibl. 15); the 102 stations from Hirvonen's catalogue are pendulum stations.

The gravity material used for the *B*-field obviously does not correspond in all parts to presentday conditions. But I have not considered it necessary to collect all the series of gravimetrical measurements of the last ten years, since most of them would not render the field more accurate as a whole for our purpose. There are still large



Map I

The f_m-field of area A.

empty areas to be filled up by gravity stations before more dense and accurate local gravimetrical surveys at greater distances from our country can be profitably utilized.

From the separate *a*-values, an *a*-field has now been constructed according to the following principles: All *a*-values within the 2 -square round every point of intersection between 1 -parallels and 1 -meridians have been averaged and the result has been referred to the »gravity» centres of the corresponding stations. Between the values of these centres isanomaly curves have been drawn by approximately linear interpolation.

From this *a*-field, characterized by its isanomaly curves, mean free air anomalies have been calculated according to the expression $f_m = a_m + 0.06\ h_m$ for landsquares of the size 1° long × 1° lat, and to the expression $f_m = a_m - 0.02\ d_m$ for sea-squares of the same size. a_m has been evaluated from the *a*-field described above, h_m or d_m is the mean height or depth of the square. If the square contains both land and sea areas we have $f_m = a_m + 0.06\ q_{land} \cdot h_m - 0.02\ q_{sea} \cdot d_m$, q_{land} and q_{sea} being the respective ratios of the land area and the seaarea to the whole area of the square. h_m and d_m have in most cases been taken from maps of the scale 1: 1 500.000, giving heights as point values and depths both as distinct values and as isohypses. (bibl. 1). h_m has been calculated in the following manner: each square was divided into »plateau»—areas, each one characterized by equally spaced pointheight-values for mountain or hill tops and valleys, the number of tops and valleys being about the same. The mean height of the »plateau»-area was computed from the expression $\frac{4\ h_v + h_t}{5}$, h_v and h_t being the mean heights of valley-points and hilltops respectively. The expression is based on the assumption that the value of h is representative for the valley-area and that the mean height of a mountain mass can be taken equal to $h_v + 0.4\ (h_t - h_v)$, the factor 0.4 being chosen instead of 1/3, which corresponds to a purely conical mountain. From the mean heights of the »plateau»-areas the mean height h_m is calculated as a weighted mean of the results for the areas, the weights being equal to the magnitudes of the areas. Where possible, 16 height-values have been used for every 1′-square. Checking this method of calculation upon some squares in the *A*-area with mean heights between 90 and 350 mts, we find a standard error of about ± 10 mts, and no tendency of correlation between the magnitudes of the separate errors and the mean heights can be detected. For a few squares of the area *B* with large mean heights and great variations in height the abovementioned calculation may be inaccurate if we do not take account of more than 16 height-values. But in general such topography can be studied on maps of greater scales, which contain a sufficient number of height-values. To obtain some information of the resulting accuracy for such squares, when using the 16-value density, I have compared the results for several 1′-squares in Switzerland (mean height ~ 1.500 mts) with the corresponding results from 64 heights (map scale 1: 750.000). It is found that the differences between the results do not exceed 10 p.c. of the mean height in any one of the investigated squares. Using 16 height-values for 1′-squares with h_m < 750 mts and 64 values for 1′-squares with greater mean height (alpine squares), I believe that we can always rely upon an accuracy for h, which corresponds to a standard error of about ± 0.05 h_m.

d_m for the sea regions has been evaluated, as a rule, from point-values *and* isohypses on the same maps as h. In a few cases only (squares over the great depths in the *Norwegian sea* and the *Barents sea*) two French marine charts have been taken into account (bibl. 3a, b).

In constructing the a-field, no account has been taken of the great orographic corrections within some mountain regions (especially the Alps). Therefore, in order to obtain a more representative cogeoidal field for integration than the free air one, I have proceeded as follows:

It can easily be seen from the table I (see the rules for use in ch I) that a valley-station gravity requires a reduction to the cogeoid, which is approximately equal to the free air reduction + *twice the terrain-correction.* For a mountain-station gravity, on the other hand, the reduction effect of topography and its »reflection» contains the *difference* between the attraction on P from the orographic masses at the station level and the corresponding attraction on P_o from the same masses at the reflection level of P. This part of the reduction effect, therefore, may be neglected in comparison to the great corresponding valley-station effect. Considering all topographical features within an 1°-square, and bearing in mind that gravity stations are in general valley-stations, an average value of the orographic corrections for stations within the square (known from the catalogues and reports) may be added to the mean free air anomaly in order to obtain a more representative cogeoidal one.

The orographic correction, however, depends both upon the steepness of the topographical contours and the amplitude of the height-variation. If the amplitude is great and when the average distance between consecutive mountain tops and valleys is small, the orographic correction will be considerable. Taking known corrections for several squares within the area B, we may assume them to be correlated with the ratio $\varkappa = \frac{a}{s}$, where a denotes the average amplitude, s the average distance between mountain tops and valley-centres within the square. The correlation can be illustrated in a diagram. The quantities $\varkappa$ have been calculated for all squares at the same time as h_m, and from the diagram the correction ($K_m^{(co)}$) can be taken. For most 1°-squares $K_m^{(co)}$ is negligible but for some squares in the Alps it may exceed + 20 mgals. After having corrected f_m by $K_m^{(co)}$, we obtain mean cogeoidal anamalies for every 1°-square.

Through approximate linear interpolation between these anomalies, which have been referred to the centres of the squares, we obtain the anomaly-field of the area B, which has been used by the integrations included in chs VI and VII (see map II).

Comparing this B-field to the f_m-field of the area A, we obtain differences between the latter and the former fields for a number of squares of about 3.000 sqkms. Regarding these differences as independent and normally distributed, we obtain a standard error (m_0) for one square equal to ± 6 mgals. From this result I have concluded that in such parts of the B-field, where the mean height for 1°-squares < 400 mts and where the station density $^{1\ p.}/_{1.200\ sqkms}$ has been used by the construction of the cogeoidal field, the mean accidental error (m) of an integration compartment (T sqkms) will be

$$m \sim \pm 6 \sqrt{\frac{3.000}{T}} \text{ mgals}$$

if we assume the mean error of cogeoidal representation to be neglected. The last assumption will be justified if the average terrain correction (orographic correction) over the compartment is small (say $< \pm 3 \sqrt{\frac{3.000}{T}}$ mgals). With regard to the uncertainty due to abnormal local curvatures, we have pointed out earlier (ch I) that it may certainly be neglected outside the *A*-field. Since, if we again assume the mean error of cogeoidal representation due to this cause equal to $\pm$ 2 mgals per 50 sqkms, we should have $m = \pm$ 0.3 mgals for the smallest integration compartment within the *B*-field.

Under the above-mentioned conditions the *B*-field would contribute the following mean errors in the different integration compartments the decimals of m being only formal:

a) *Deflections.*

Zone	T sqkms	$\sqrt{T/3.000}$	m	n	m_E
			$\pm$		$\pm$
13	4300	1.2	5.0	3.7	5.4
14	8370	1.7	3.5	7.2	5.0
15	13650	1.2	1.7	11.8	4.6
16	24500	2.9	2.0	21.1	3.9
17	37100	3.6	1.6	32.0	3.5
18	60800	4.6	1.2	52.4	2.9

b) *Curvatures.*

Zone	T sqkms	$\sqrt{T/3.000}$	m	n	m_E
			$\pm$		$\pm$
13 c	3450	1.1	5.5	3.0	5.5
13 d	31200	3.5	1.7	26.9	3.7
13 e	112000	6.2	1.0	97	2.3

Here m_E is the mean error of representation for the mean free air anomaly in the compartment, calculated from Hirvonens E-curve, *n* being the number of measured anomalies according to the aforesaid density $^{1\ p.}/_{1.200\ sqkms}$. The above-mentioned assumptions, however, on which the tables a) and b) are based will only be realized within restricted areas of the *B*-field. The station density for land-areas varies from $^{1\ p.}/_{1.200\ sqkms}$ to $^{1\ p.}/_{80.000\ sqkms}$ and the accuracy of $+ 0.06\ (h_m - h_i) + K_m^{(co)}$, involved in the mean cogeoidal anomalies taken from map II alters from one part to another within the *B*-field. The density for open sea-regions is less than $^{1\ p.}/_{180.000\ sqkms}$.

By the calculation of mean errors in curvatures and deflections I have not hitherto regarded it as neccessary to carry out a detailed investigation of the accuracy for various parts of the *B*-field. As mentioned on p. 29 the real variation of b_{land} and b_{sea} is not yet known and before obtaining a clearer picture of this variation I have only roughly

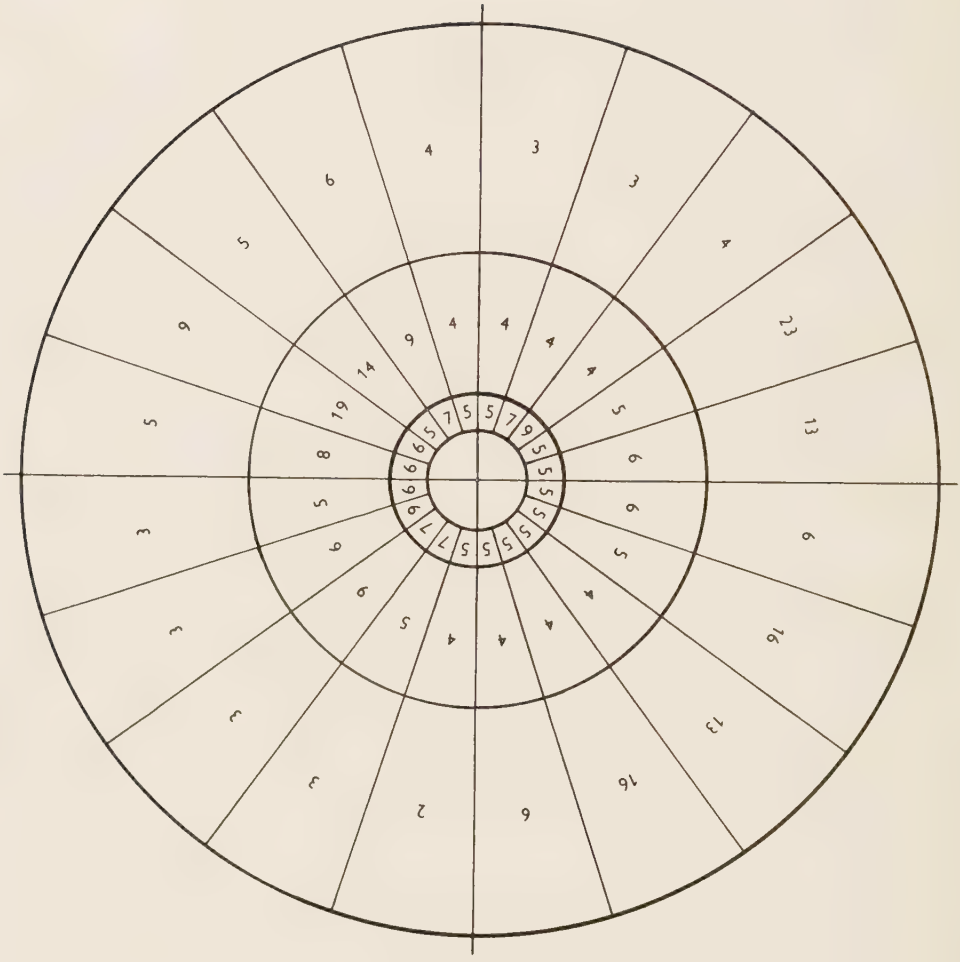


Fig. 6 a.

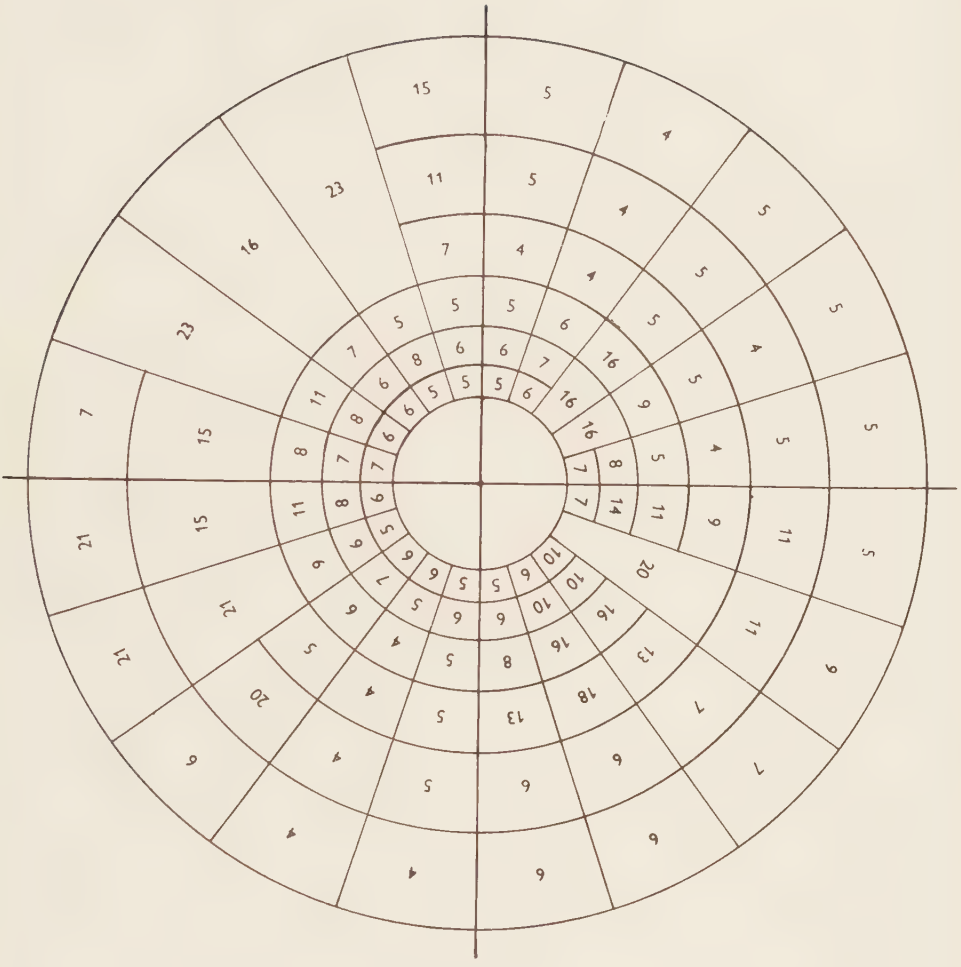


Fig. 6 b.

estimated the contribution of mean error from the *B*-field, considering m_E calculated from Hirvonens error of representation for level land, as an upper limit. In fig. 6 these values of m_E for all compartments used by the integrations for a) curvatures and b) deflections are written down as they ocur in the integration template.

In the discussion of accuracy above I have neglected the measuring errors and the errors due to the possible inaccuracies in the values of the reference stations.

The C-field.

This field has been constructed from mean free air anomalies for 10°-squares, investigated by Jeffreys in his interesting study of the earth's gravitational field. (bibl. 10) The mean anomalies are freed from the systematic error $-b\,(h_m - h_i)$ according to the same equation of linear regression $f = a + b\,h$, which has been used for our *A*- and *B*-fields. The values of b_{land} and b_{sea} are taken from separate investigations concerning the different 10°-squares. For such squares where b could not be determined, the world medians (see p. 22) have been accepted. h_m and d_m have been interpolated from a list prepared by Prey. The gravity material, the main part of which concerns the Northern hemisphere, contains about 3.000 stations, chiefly emanating from Heiskanen's catalogue of isostatic anomalies (bibl. 8).

Jeffreys mean anomalies have, in this paper, been treated as cogeoidal ones, and the first part of the *C*-field has been constructed with its isanomaly curves by approximately linear interpolations between the original mean anomaly-values, inserted at the points of intersection for 10°-parallels and 10°-meridians. Rice in his important study of deflections of the vertical (bibl. 16) also makes use of Jeffreys' investigations, but he simply takes the free air anomalies calculated from Jeffreys' definite solution for the earth's gravity (expressed on the Potsdam standard). As this solution does not contain higher spherical harmonics than those of the third order, it will describe the variations of gravity (and anomalies) only in very broad trends. In Rice's case (investigations in the U.S.A.), it was the best field available as regards the very low gravity density in the Southern Pacific, but for investigations of deflections in Europe it would be inconvenient and unnecessary to smooth out the whole field to this high degree.

For that reason I have used the original 10°-results of Jeffreys for the *land hemisphere*, interpolating between them without taking account of their varying accuracy. Our deflection-points (see chs VI and VII) are situated close to the centre of this hemisphere. For the *water hemisphere*, on the other hand, Jeffreys definite solution has been directly utilized to obtain a cogeoidal field.

In order to decide the degree of accuracy obtainable by using the *C*-field when we integrate for deflections and curvatures, I have calculated m_E (see above) for all compartments of integration, which contain at least one gravity station. If there are empty *unit* compartments (radial contribution per mgal 0."01 and $0."01/_{10\ kms}$ respectively), they have been taken together with the nearest occupied compartment of the same azimuth in order to include at least one station within the whole aggregate. The table shows (p 40) such complexes of compartments, each one containing its value of m_E. The mean anomalies of these combined compartments have been regarded as independent.

A comparison between the first part of the *C*-field (field of land hemisphere) and the *B*-field, described before, gives the following information relating to independence and accuracy, when the station density used by Jeffreys is greater than $^{1}\,p./_{70.000\ sqkms}$: For 56 squares, each one covering 1/9 of the area of one 10°-square, the differences of cogeoidal anomalies between the *B*- and *C*-fields have been evaluated. The means for runs of signs (horizontal and vertical) is 2. Thus, I have considered areas of the magnitude $\frac{4}{9}$ × one 10°-square at 55° lat. as independent and for such areas we obtain from the differences (neglecting the errors of the *B*-field) the standard error ± 6 mgals. The mean station density in the part of the *C*-field considered is $^{3}\,p./_{70.000\ sqkms}$ (minium density $^{1}\,p./_{70.000\ sqkms}$, maximum density $^{1}\,p./_{10.000\ sqkms}$). An average »independent square» of the type mentioned above subtends an area of about 300.000 sqkms and contains about 13 gravity stations. For such a square m_E would be ± 6.7 mgals.

The anomaly fields have been used for the integration in the following manner. The integration template was drawn with its 20°-sectors and zones of unit contribution with the aid of the geographical coordinates for the points of intersection between zonal circles and sector radii. These points have been calculated on the assumption that $\varphi = 60° N$, $\lambda = 15° E$ is the centre of the template. It can be shown that this template may be utilized for all computations concerning the area investigated for deflections and curvatures (chs VI and VII). The world maps, on which the *C*-field and the template have been plotted, are constructed as azimuthal equal-area projections for the two hemispheres.

Table of m_E for deflections (A) and curvatures (B) in the C-field.

A

Zone / Sec	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
10^c	13	12	17	17	25	26		26	27			9						
30	15	23	17	24	25	25	25	26	19			16				19		
50	11	13	25		25	25	15	26	18	26		9			27	19	13	
70	11	13	14	11	10	9	8	6	9	9		7	4	9		19	26	
90	15	13	14	9	10	9	9	4	6	6		5	3	5	16			
110	9	12	11	9	9	7	6	5	4	3		4	5	9	8	27		
130	11	12	11	7	7	6	6	10	8	6		28						
150	15	16	17	8	7	6	7	10	13	19		8						
170	8	9	10	9	9	11	10	10	8	13		8	19					
190	8	9	10	7	7	18	11	26	18	19		5	14					
210	11	10	9	5	5	25	25	27				7	28					
230	8	7	11	5	5	7	18	19				12	11	28				
250	8	9	7	6	4	5	13	10	13	8		7	8	10	13	16		
270	9	9	12	17	17	8	9	10	11	13		11	11	9				
290	15	13	17	24	9			9	26	26		20						
310	22	16	14	10								10						
330	15	16	24	26					8	4		8						
350	25			24	27					5		14						
370	25			24	25	25	25	18	18	8		13	27					
390	22	23	25		25	25	25	26	26	19		13	8					
Contr.	+ 0.01	+ 0.01	+ 0.01	− 0.01	+ 0.01	+ 0.01	+ 0.01	+ 0.01	+ 0.01	+ 0.01	0	- 0.01	− 0.01	- 0.01	− 0.01	- 0.01	− 0.01	0

B

(Only zone 14)

10^c	30	50	70	90	110	130	150	170	190	210	230	250	270	290	310	330	350	370	390
5	5	3	2	2	1	3	3	3	2	3	2	2	3	4	2	2	2	3	5

III.

Formulas for deflections of the vertical.

According to Vening Meinez we obtain from Stokes' formula for the cogeoidal elevation (N), through derivation

$$-\frac{\partial N}{\partial x} = \xi_{\text{rad}} = \varphi_{\text{astr}} - \varphi_{\text{geod}} = +\frac{1}{4\pi G}\int_0^{2\pi}\int_0^{\pi} \cos\alpha \sin\psi \frac{df}{d\psi} \triangle g \, d\alpha \, d\psi$$

$$-\frac{\partial N}{\partial y} = \eta_{\text{rad}} = (\lambda_{\text{astr}} - \lambda_{\text{geod}})\cos\varphi = +\frac{1}{4\pi G}\int_0^{2\pi}\int_0^{\pi} \sin\alpha \sin\psi \frac{df}{d\psi} \triangle g \, d\alpha \, d\varphi \qquad \text{III: 1}$$

Here α and ψ represent the azimuth and angular distance for a point with the anomaly $\triangle g$ on the cogeoid, α being reckoned from the north (direction of x-axis), ψ being measured from the station P_0 for which ξ and η are to be calculated. The integration should be performed for the whole earth. G is the mean gravity of the earth in mgals and f the wellknown function of Stokes:

$$f = \operatorname{cosec}\frac{\psi}{2} + 1 - 6\sin\frac{\psi}{2} - 5\cos\psi - 3\cos\psi \ln\left(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}\right)$$

$$\text{with } \frac{df}{d\psi} = -\frac{1}{2}\cot\frac{\psi}{2}\left[\operatorname{cosec}\frac{\psi}{2} + \frac{3}{1+\sin\frac{\psi}{2}} + 12\sin\frac{\psi}{2} - 32\sin^2\frac{\psi}{2} - 12\sin^2\frac{\psi}{2}\ln\left(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}\right)\right] \qquad \text{III: 1a}$$

In the formula III: 1 ξ is positive when the cogeoid descends towards the north, η is positive when it descends towards the east (λ is reckoned from Greenwich eastwards), φ and λ are in radians.

From radians we go over to centesimal seconds of an arc and obtain when introducing

$$p = \sin\frac{\psi}{2}$$

$$-\xi = -0.1033\int_0^1 \sqrt{1-p^2}\left[\frac{1}{p} + \frac{3}{1+p} + 12p - 32p^2 - 12p^2\ln p(p+1)\right]\int_0^{2\pi}\cos\alpha \triangle g \, d\alpha$$

$$-\eta = -0.1033\int_0^1 \sqrt{1-p^2}\left[\frac{1}{p} + \frac{3}{1+p} + 12p - 32p^2 - 12p^2\ln p(p+1)\right]\int_0^{2\pi}\sin\alpha \triangle g \, d\alpha \qquad \text{III: 2}$$

For $\psi = 0$, the integrand in $\int_0^1$ becomes discontinous. Breaking up the integral into two parts and taking

$$0.1033 \sqrt{1-p^2}\left[\frac{1}{p} + \frac{3}{1+p} + 12\,p - 32\,p^2 - 12\,p^2 \ln(p+p^2)\right] = F(p)$$

we obtain

$$\xi'' = -\int_0^{p_0} F(p)\,dp \int_0^{2\pi} \cos\alpha \triangle g\, d\alpha - \int_{p_0}^{1} F(p)\,dp \int_0^{2\pi} \cos\alpha \triangle g\, d\alpha$$

$$\eta'' = -\int_0^{p_0} F(p)\,dp \int_0^{2\pi} \sin\alpha \triangle g\, d\alpha - \int_{p_0}^{1} F(p)\,dp \int_0^{2\pi} \sin\alpha \triangle g\, d\alpha \qquad \text{III: 2a}$$

The last integrals can be integrated directly, the first one can be proved to have a limit, and if ψ_0 is small so, that $p_0 = \sin\frac{\psi_0}{2} \sim \frac{\psi_0}{2}$, this limit can be deduced in the following manner: assume that the field $\triangle g$ within a circle of angular radius ψ_0 can be written

$$\triangle g = \triangle g_0 + \left(\frac{\partial \triangle g}{\partial x}\right)_0 dx + \left(\frac{\partial \triangle g}{\partial y}\right)_0 dy + \quad \ldots\ldots\ldots \text{III: 3}$$

Where $dx = a\,\psi\cos\alpha$; $dy = a\,\psi\sin\alpha$, a being the mean radius of the earth. We have (denoting the derivatives by k_x, k_{xx} etc)

$$\int_0^{2\pi} \cos\alpha \triangle g\, d\alpha = \int_0^{2\pi} (\triangle g_0 + k_x\,dx + k_y\,dy + \ldots)\cos\alpha\, d\alpha =$$

$$= \int_0^{2\pi} (k_x\,a\,\psi\cos\alpha + k_y\,a\,\psi\sin\alpha + \frac{1}{2}k_{xx}\,a^2\,\psi^2\cos^2\alpha + \ldots)\cos\alpha\, d\alpha \quad ;$$

Here all integrals but the first one become zero, and we have

$$\int_0^{2\pi} \cos\alpha \triangle g\, d\alpha \sim \pi\, k_x\, a\,\psi$$

$$\int_0^{2\pi} \sin\alpha \triangle g\, d\alpha \sim \pi\, k_y\, a\,\psi$$

if we assume the field to have constant derivatives of the second order within the circle ψ_0.

We now have

$$\xi_0'' - \int_0^{p_0} a \cdot 2\, p\, F(p)\, dp \cdot \pi \cdot k_x = -\, 2\, a\, \pi\, k_x \int_0^{p_0} p\, F(p)\, dp$$

$$\eta_0'' = -\, 2\, a\, \pi\, k_y \int_0^{p_0} p\, F(p)\, dp$$

Take $\sqrt{1 - p^2} = 1 - \frac{1}{2} p^2$ and include in the integrand only the terms up to the first order:

$$\int_0^{p_0} p\, F(p)\, dp = 0.1033\, p_0 + 0.1550\, p_0^2$$

Taking $p_0 = \frac{\psi_0}{2} = \frac{\delta s}{2a}$, we get

$$\xi_0'' = -\, 0.1033\, \pi\, k_x\, \delta s - \frac{\delta s^2}{2a}\, \pi\, k_x \cdot 0.1550 =$$

$$-\, 0.325\, k_x\, \delta s_{km} \left[1 + 1.2 \cdot 10^{-4}\, \delta s_{km} \right]$$

when $k_x = \left(\frac{\partial \triangle g}{\partial x} \right)_0$ expressed in mgal/km

Taking, for instance, $\delta s = 32$ kms, the maximum radius used in ch VI, we thus introduce of about 0.4 %, if we use the formulas below for ξ_0 and η_0

$$\xi_0'' = -\, 0.325 \left(\frac{\partial \triangle g}{\partial x} \right)_0 \cdot \delta s_{km}$$

$$\eta_0'' = -\, 0.325 \left(\frac{\partial \triangle g}{\partial x} \right)_0 \cdot \delta s_{km} \qquad \text{III: 3 a}$$

The last integrals were

$$\xi'' - \xi_0'' = -\int_{p_0}^{1} F(p)\, dp \int_0^{2\pi} \triangle g \cos \alpha\, d\alpha$$

$$\eta'' - \eta_0'' = -\int_{p_0}^{1} F(p)\, dp \int_0^{2\pi} \triangle g \sin \alpha\, d\alpha \qquad \text{III: 4}$$

For the integration compartment, indicated in fig. 7, the contribution to $\xi'' - \xi_0''$ is

$$\int_{p_{n-1}}^{p_n} F(p)\, dp \int_{\alpha_{k-1}}^{\alpha_k} \triangle g \cos\alpha\, d\alpha$$

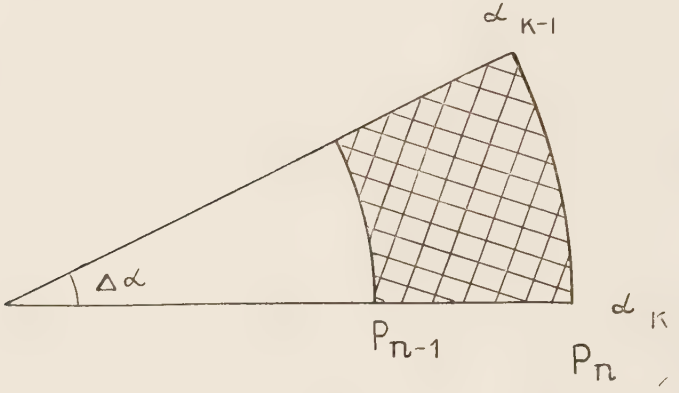


Fig. 7.

Chosing the aperture of the compartment equal to 20^c, we obtain for the effect in a radial direction per mgal of $\triangle g$ in the compartment

$$\frac{\pi}{10} \int_{p_{n-1}}^{p_n} F(p)\, dp = 0.314 \int_{p_{n-1}}^{p_n} F(p)\, dp$$

Fixing this contribution at $0.''01$, we get

$$\int_{p_{n-1}}^{p_n} F(p)\, dp = 3.18\ 10^{-2}$$

Beginning with a least circle of 5 km radius ($p_0 = 3.92 \cdot 10^{-4}$), I have calculated $F(p)$ for different values of p from p_0 to 1. From these $F(p)$-values, plotted as curves, I have *graphically* determined $\int_{p_0}^{p}$ for different values of p. We are now able to compute p_1, p_2, etc, so that the zone $p_{pn1} \rightarrow p_n$ always contributes $0.''01$ per mgal and compartment. See the curves in fig. 8 a, b, c and the zone table on the next page.

Zone N°	Outer radius km	Contr./ comp. 1 mgal	Zone N°	Outer radius km	Contr./ comp. 1 mgal	Zone N°	Outer radius km	Contr./ comp. 1 mgal
	5.0							
1	6.9	+ 0."01	13	255	+ 0."01	25	3870	+ 0."01
2	9.3	+ 0."01	14	344	+ 0."01	26	4470	+ 0."01
3	12.5	+ 0."01	15	453	+ 0."01	27	5240	+ 0."01
4	17.2	+ 0."01	16	601	+ 0."01	28	6290	+ 0."01
5	23.3	+ 0."01	17	773	+ 0."01	29	9500	± 0."00
6	31.6	+ 0."01	18	990	+ 0."01	30	10900	− 0."01
7	42.7	+ 0."01	19	1260	+ 0."01	31	12000	− 0."01
8	58.6	+ 0."01	20	1620	+ 0."01	32	12900	− 0."01
9	78.9	+ 0."01	21	1980	+ 0."01	33	14100	− 0."01
10	106	+ 0."01	22	2400	+ 0."01	34	15600	− 0."01
11	144	+ 0."01	23	2810	+ 0."01	35	19200	− 0."01
12	194	+ 0."01	24	3340	+ 0."01	36	ant.	0

Zone—table for deflections.

When integrating for absolute deflection (ch VII) I have used
the Wideland field (A) for the zones 1—12,
the European field (B) for the zones 13—18,
and the Globular field (C) for the zones 19—36. (see ch II).

In this deduction of deflection zones I have not used the tables of Sollins (bibl. 8), because for the calculation of curvature zones (see ch IV), which was made at the same time, no tables for $\frac{d^2 f}{d \psi^2}$ exist. The results of deflection zone radii, however, have been checked with the aid of Sollins' tables.

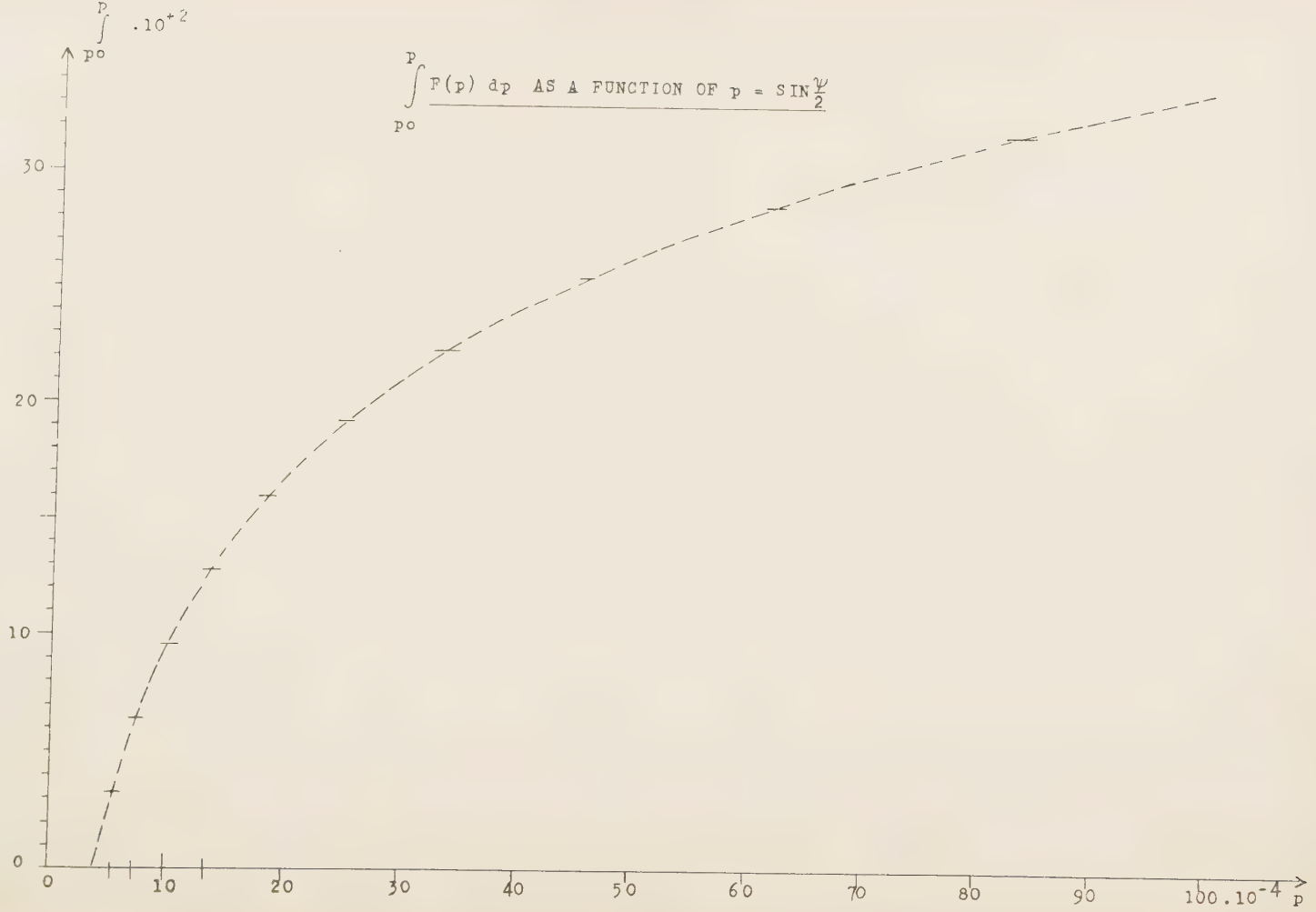

$\int_{p_o}^{p} .10^{+2}$
$\int_{p_o}^{p} F(p)\,dp$ AS A FUNCTION OF $p = \sin\frac{\psi}{2}$
0
10
20
30
0
10
20
30
40
50
60
70
80
90
100.10^{-4} p

Fig. 8 a.

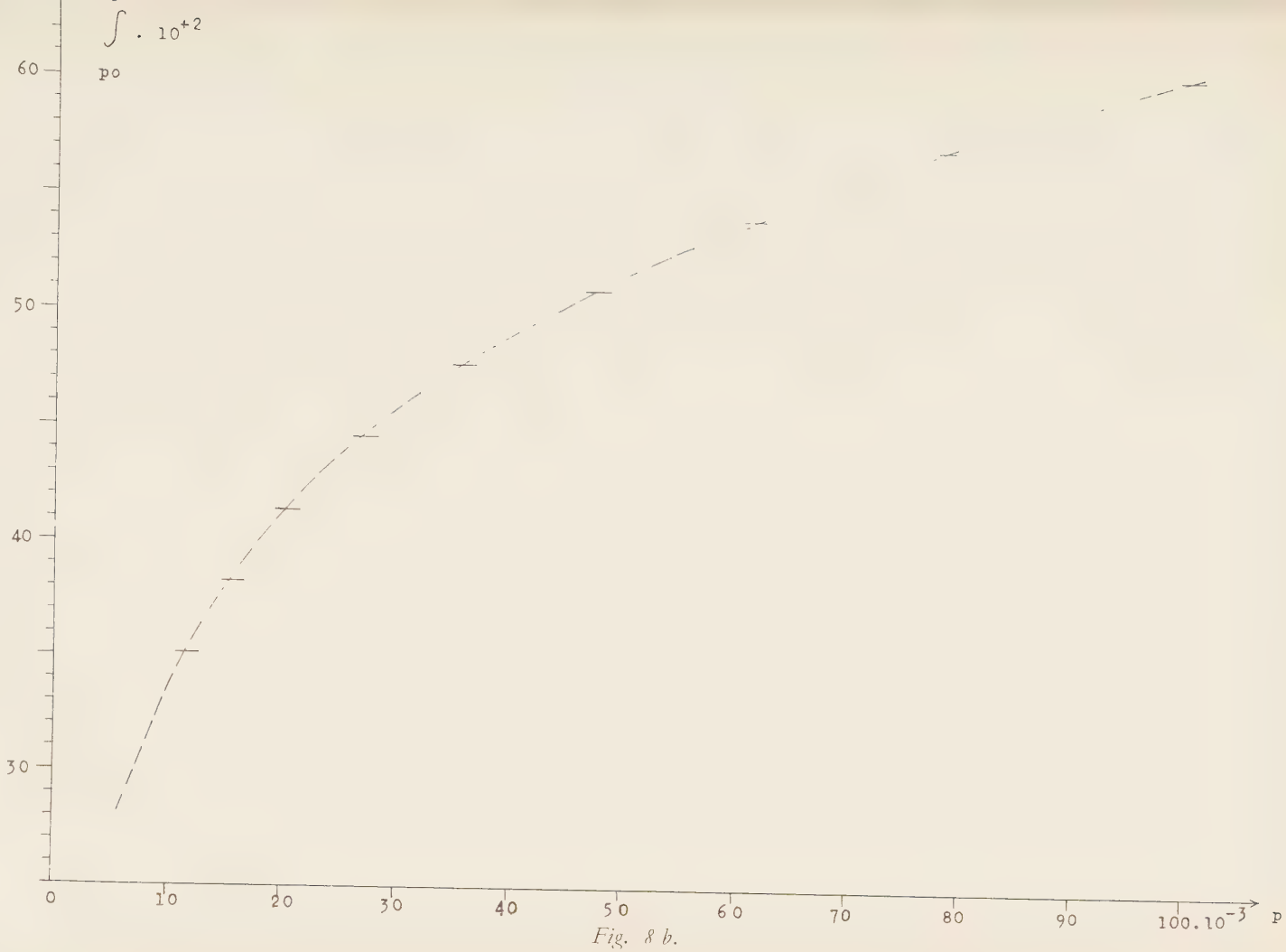


Fig. 8 b.

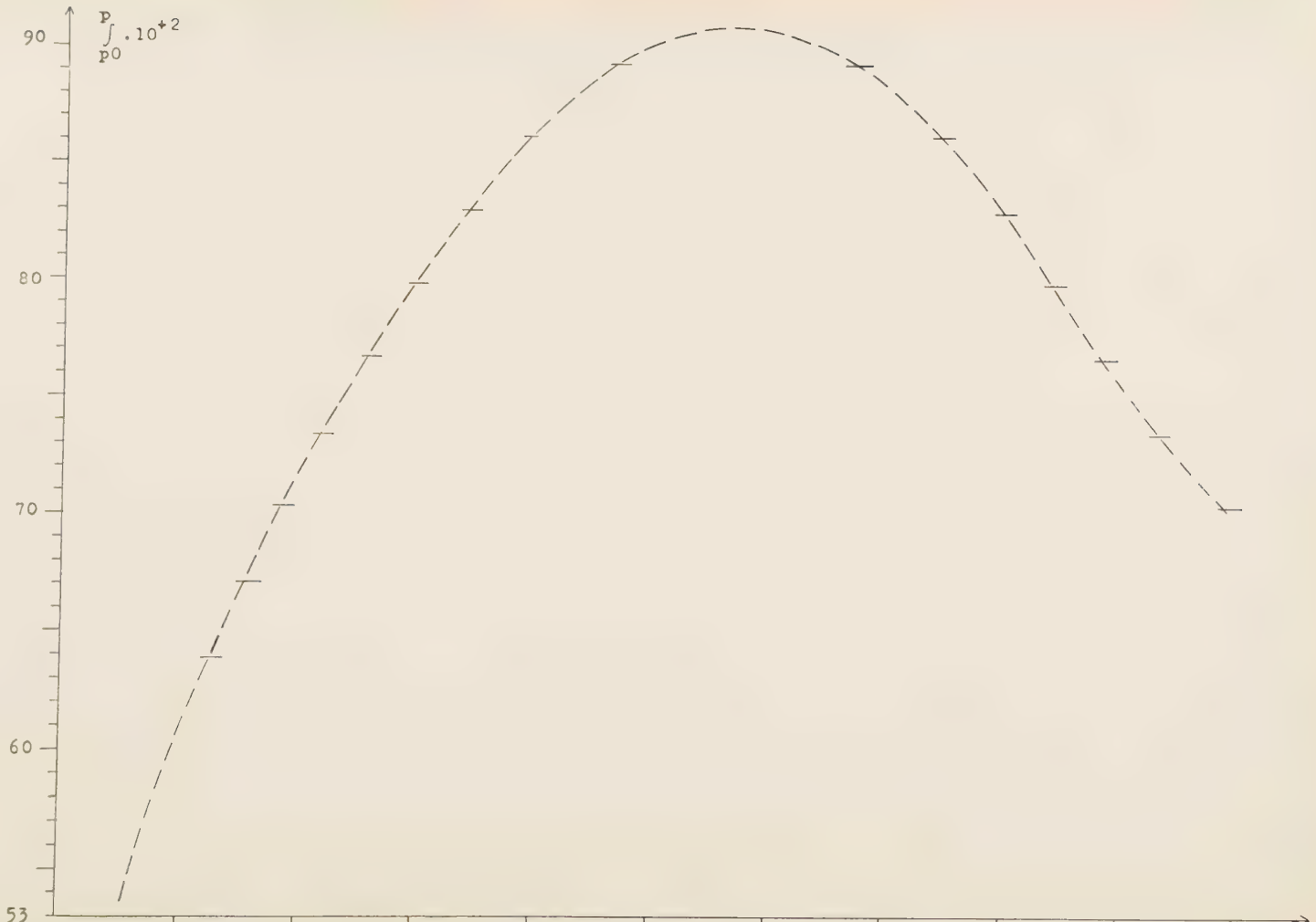


Fig. 8 c.

IV.

Formulas for cogeoidal curvatures obtained from Stokes' formula.

In order to investigate the structure of the geoid over restricted areas without requiring to determine its absolute angular orientation and N-values with respect to the international ellipsoid of reference (see ch VI), I have used the second derivatives $\frac{\partial^2 N}{\partial x^2}$, $\frac{\partial^2 N}{\partial x \partial y}$ and $\frac{\partial^2 N}{\partial y^2}$ derived from Stokes' formula to obtain the relative undulations of the cogeoidal surface. It can be shown (see further below) that these »components of curvature» ordinarily depend very little upon the gravity field beyond a distance of 100 kms from the point at which the curvature is required. Thus, for such parts of a fairly dense and homogeneous gravity net, which are situated within a line at a distance of 100 kms from the front of sparsely filled or empty gravity regions, curvatures can be calculated only taking the gravity field outside the 100 km line very roughly into consideration.

The gravity field over an area the central part of which has to be investigated according to its geoidal structure, has a given average station density which may be used to define a generalized cogeoidal field and à corresponding cogeoid (see chs II, VI). As regards our area of investigation within the central part of the A-field, this station density amounts to about $^{1\ P.}/_{250\ sqkms}$ and the generalization here has been characterized chiefly by the construction of a 2nd degree field (chs II, VI) within circles of 32 kms radius round the curvature-points. As the density for these points I have chosen $^{1\ P.}/_{200\ sqkms}$, and this corresponds to the calculation of curvature-components for all corners of $^1/_8{}^\circ$ long $\times$ $^1/_4{}^\circ$ lat-squares, which are situated inside the above-mentioned line, at a minimum distance of 100 kms from the limits of the A-field. Each point gives a triplet of curvature-components

$$\left(\frac{\partial^2 N}{\partial x^2} = -\frac{\partial \xi}{\partial x}, \frac{\partial^2 N}{\partial x \partial y} = -\frac{\partial \xi}{\partial y} = -\frac{\partial \eta}{\partial x}, \text{ and } \frac{\partial^2 N}{\partial y^2} = -\frac{\partial \eta}{\partial y}\right)$$

Taking all points into consideration we can illustrate three »fields», graphically plotted as curves for equal values of $\frac{\partial \xi}{\partial x}$, $\frac{\partial \xi}{\partial y} = \frac{\partial \eta}{\partial x}$ and $\frac{\partial \eta}{\partial y}$ respectively, the curves being drawn through interpolation between the calculated component-values.

From a central point P_0 within the investigated area, having the initial values ξ_0, η_0 of absolute deflections, we may obtain the deflections for an arbitrary point P through the integrations:

$$\xi_p = \xi_o + \int_o^{x_p} \frac{\partial \xi}{\partial x}(x,\ o)\ dx + \int_o^{y_p} \frac{\partial \xi}{\partial y}(x_p, y)\ dy$$

$$\eta_p = \eta_o + \int_o^{x_p} \frac{\partial \eta}{\partial x}(x,\ o)\ dx + \int_o^{y_p} \frac{\partial \eta}{\partial y}(x_p, y)\ dy \qquad \text{IV: 1}$$

$(o;\ o)$, $(x_p;\ y_p)$ are the coordinates of P_0 and P respectively, x and y being reckoned along the meridian and parallel through P to the points of intersection with the parallel and meridian through P_0. The integrations have been carried out graphically.

The expressions for ξ and η on p. 41 must be derived with respect to x and y, if we wish to deduce the »curvature-components». The results arrived at for $\frac{\partial \xi}{\partial x}$ and $\frac{\partial \eta}{\partial y}$ are given by de Graaf-Hunter (bibl. 6). To demonstrate the deduction and to obtain a formula for $\frac{\partial \xi}{\partial y}$ also, I proceed as follows. Considering two deflection-stations P, P' along the same parallel, we can form an expression for $\triangle y$, $\triangle \xi$ being defined as $\xi' - \xi$, namely, the *small* difference between the values of ξ in P' and P. To obtain $\frac{\partial \xi}{\partial y}$, we have to make $\triangle y \to o$ that is $\alpha' \to \alpha$ and $\psi' \to \psi$. The following relations can be taken from the figure:

$$\begin{aligned} \sin\psi \cos\alpha &= \sin\psi' \cos\alpha' \\ \cos\psi' &= \cos\triangle y \cos\psi + \sin\triangle y \sin\psi \sin\alpha \\ \cos\psi &= \cos\triangle y \cos\psi' - \sin\triangle y \sin\psi' \sin\alpha' \end{aligned} \qquad \text{IV: 2 a}$$

Moreover, as $\triangle y^{rad}$ is small, we have from the two last equations

$$\sin\psi \sin\alpha = \sin\psi' \sin\alpha' + \varepsilon_1$$

$$\cdot\ \cdot\ \cdot \qquad \frac{\cos\psi' - \cos\psi}{\sin\psi \sin\alpha}(1 + \varepsilon_2)$$

where ε_1 and ε_2 are small quantities, which will disappear at the limit. When we introduce α and ψ as independent variables instead of α' and ψ', we obtain from an elementary triangle the contribution in $\frac{\xi}{\triangle y}$

$$\frac{1}{4\pi G} \frac{\cos\alpha \sin\psi\ \triangle g\left(\frac{df}{d\psi} J - \frac{df}{d\psi}\right) d\psi\, d\alpha}{\frac{\cos\psi' - \cos\psi}{\sin\psi \sin\alpha}(1 + \varepsilon_2)} \qquad \text{IV: 2 b.}$$

where the Jacobian $J = \frac{\sin\psi \cos\psi}{\sin\psi' \cos\psi'} + \varepsilon$, ε disappearing at the limit.

Thus for $\triangle y \to o$ and $\triangle \xi \to o$, we get in $^{rad}/_{rad}$

$$-\frac{\partial \xi}{\partial y} = \frac{1}{4\pi G}\int_{o}^{\psi} \sin\psi \frac{d^2 f}{d\psi^2} d\psi \int_{o}^{2\pi} \triangle g \sin\alpha \cos\alpha \, d\alpha \qquad \text{IV: 3}$$

the same expression being obtained for $-\frac{\partial \eta}{\partial x}$.

The second derivative of f can be written:

$$\frac{d^2 f}{d\psi^2} = \frac{1}{4}\frac{2+5p+2p^2-4p^3}{p^3(p+1)} + 11 + 6p - 22p^2 + 3(1-2p^2)\ln(p+p^2)$$

where $p = \sin\frac{\psi}{2}$.

For $\psi = o$, $\frac{d^2 f}{d\psi^2}$ becomes infinite and the integrand discontinuous. This inconvenience can be avoided (see de Graaf-Hunter: bibl. 6) by subtracting from all anomalies $\triangle g$, a quantity $\triangle g_0$, equal to the anomaly at P. The formula for $\frac{\partial \xi}{\partial y} = \frac{\partial \eta}{\partial x}$ will then be

$$\frac{\partial \xi}{\partial y} = -\frac{\partial \eta}{\partial x} = \frac{1}{4\pi G}\int_{o}^{\delta\psi} \sin\psi \frac{d^2 f}{d\psi^2} d\psi \int_{o}^{2\pi} (\triangle g - \triangle g_0)\sin\alpha\cos\alpha\, d\alpha + \\ + \frac{1}{4\pi G}\int_{\delta\psi}^{\pi} \sin\psi \frac{d^2 f}{d\psi^2} d\psi \int_{o}^{2\pi} (\triangle g - \triangle g_0)\sin\alpha\cos\alpha\, d\alpha \qquad \text{IV: 4}$$

the angular distance $\delta\psi$ being small ($\sin\delta\psi \sim \delta\psi$).

The expressions for $\frac{\partial \xi}{\partial x}$ and $\frac{\partial \eta}{\partial y}$ are obtained by changing $\sin\alpha\cos\alpha$ to $\cos^2\alpha$ and $\sin^2\alpha$ respectively. We have for instance (in $^{rad}/_{rad}$)

$$\frac{\partial \xi}{\partial x} = \frac{1}{4\pi G}\int_{o}^{\delta\psi} \sin\psi \frac{d^2 f}{d\psi^2} d\psi \int_{o}^{2\pi} (\triangle g - \triangle g_0)\cos^2\alpha\, d\alpha + \qquad \text{IV: 4 a}$$

$$+ \frac{1}{4\pi G}\int_{\delta\psi}^{\pi} \sin\psi \frac{d^2 f}{d\psi^2} d\psi \int_{o}^{2\pi} (\triangle g - \triangle g_0)\cos^2\alpha\, d\alpha$$

The second integration should be carried out directly. The first can be simplified as $\delta\psi$ is small, and it can be proved to have a limit. Let it be assumed that the anomaly-field $\triangle g$ within the circle $\delta\psi$ may be illustrated by a surface $\triangle g\,(x,y) = Z$ *of 2nd degree*

$$Z = Ax^2 + By^2 + 2Cxy + Dx + Ey + \triangle g_0$$

We then have

$$\triangle g - \triangle g_0 = \left(\frac{\partial \triangle g}{\partial x}\right)_0 dx + \left(\frac{\partial \triangle g}{\partial y}\right)_0 dy +$$

$$+ \frac{1}{2}\left\{\left(\frac{\partial^2 \triangle g}{\partial x^2}\right)_0 dx^2 + 2\left(\frac{\partial^2 \triangle g}{\partial x\, \partial y}\right)_0 dx\, dy + \left(\frac{\partial^2 \triangle g}{\partial y^2}\right)_0 dy^2\right\} =$$

$$= D\, dx + E\, dy + A\, dx^2 + C\, dx\, dy + B\, dy^2$$

where $\begin{cases} dx \approx a\, \psi \cos \alpha \\ dy \approx a\, \psi \sin \alpha \end{cases}$ ψ being $< \delta\, \psi$; a is the mean radius of the earth.

With this expression for $\triangle g - \triangle g_0$ the first integral in IV: 4 a becomes

$$\frac{a^2}{16\, G}(3\, A + B) \int_0^{\delta \psi} \left(\psi - \frac{\psi^3}{6} + \ldots\right) \psi^2 \frac{d^2 f}{d \psi^2}\, d\psi$$

or

$$\frac{a^2}{16\, G}(3\, A + B)\left(4\, \delta \psi + \frac{3}{2}\, \delta \psi^2 + \ldots\right)$$

If we wish to have $\frac{\partial \xi}{\partial x}$ in $^{m}/_{10\ \mathrm{kms}}$, we get the contribution from the first integral equal to

$$-\left(\frac{\partial \xi}{\partial x}\right)_0 {}^{m}/_{10\ \mathrm{kms}} = t_0 + 5.8 \cdot 10^{-4}\, t_0\, \delta s$$

where $t_0 = 0.162\ (3\, A + B) \cdot \delta s$, δs being in units of 10 kms (»mil»).
The error committed by neglecting the second term in $-\left(\frac{\partial \xi}{\partial x}\right)_0$ is only 0.2 % at a radius of about 30 kms.

In the second integral of IV: 4 a we introduce, as before, the variable $p = \sin \frac{\psi}{2}$ with $p_0 = \sin \frac{\delta \psi}{2}$ and obtain

$$\frac{1}{4\pi G} \int_{p_0}^{1} \left\{\frac{2 + 5\, p + 2\, p^2 - 4\, p^3}{p^2\,(p+1)} + 44\, p + 24\, p^2 - 88\, p^3 + \right.$$

$$\left. 12\, p\,(1 - 2\, p^2) \ln\,(p + p^2)\right\} dp \int_0^{2\pi} (\triangle g - \triangle g_0) \cos^2 \alpha\, d\alpha$$

For the integration compartment, shown in fig. 7, the contribution to $-\left(\frac{\partial \xi}{\partial x} \quad \left(\frac{\partial \xi}{\partial x}\right)_0\right)$ is in $^{m}/_{10\ \mathrm{kms}}$

$$8.12 \cdot 10^{-5} \int_{p_{n-1}}^{p_n} F^*(p)\, dp \int_{\alpha_{k-1}}^{\alpha_k} (\triangle g - \triangle g_0) \cos^2 \alpha\, d\alpha$$

if we denote the expression within the brackets $\{\ \}$ above by $F^*(p)$.

The aperture of the compartment sector being 20^c as before (see ch III) we obtain the radial conbribution per mgal (in $\triangle g - \triangle g_0$) equal to

$$2.55 \cdot 10^{-5} \int_{p_{n-1}}^{p_n} F^*(p)\, dp \quad \text{..... IV: 5}$$

Fixing this contribution at $0''.01/_{10\ \text{kms}}$, we can determine the zone radii of the integration template from the equation

$$\int_{p_{n-1}}^{p_n} F^*(p)\, dp = 392 \quad \text{..... IV: 6}$$

As in ch III by the calculation of deflection-zone radii I have chosen 5 km as the smallest circle radius ($p_0 = 3.92 \cdot 10^{-4}$). I have computed $F^*(p)$ for different values of p in the interval $p_0 \rightarrow 1$. From these $F^*(p)$-values, plotted as curves I have graphically determined $\int_{p_0}^{p}$ for different values of p. We are now able to calculate p_1, p_2, etc so that the zone $p_{n-1} \rightarrow p_n$ always contributes $0''.01/_{10\ \text{kms}}$ per compartment and per mgal in $\triangle g - \triangle g_0$. The integrals $\int_{p_0}^{p}$ plotted as curves (see fig. 9) show that the contribution per mgal amounts to about $0''.01/_{10\ \text{kms}}$ from $p = 0.005$ to the antipode. But $p = 0.005$ corresponds to a radius of only 60 kms. It is thus obvious that the gravity field outside, say, 100 kms should be taken into account only roughly. The radii of the curvature-zones are shown in the following table. The zone 13 has been divided into five parts (13 a—e), each one giving a contribution of $0.002''/_{10\ \text{kms}}$ per compartment and mgal. The uncertainty of the evaluated radii amounts as an average to about 1 p.c.

Zone N°	Outer radius km	Contr./ comp. 1 mgal $''/_{10\ \text{kms}}$	Zone N°	Outer radius km	Contr./ comp. 1 mgal $''/_{10\ \text{kms}}$	Zone N°	Outer radius km	Contr./ comp. 1 mgal $''/_{10\ \text{kms}}$
	5.0							
1	5.4	0.01	7	10.8	0.01	13 a	76	0.002
2	5.9	0.01	8	13.0	0.01	13 b	99	0.002
3	6.6	0.01	9	16.3	0.01	13 c	178	0.002
4	7.4	0.01	10	21.6	0.01	13 d	480	0.002
5	8.3	0.01	11	31.8	0.01	13 e	970	0.002
6	9.4	0.01	12	62.4	0.01	14	20000	0.001

Zone table for curvatures.

When integrating for curvatures in the generalized field described above, the Wideland field (A) has been used for the zones 12, 13 a, 13 b, the European field (B) for the zones 13 c, 13 d, 13 e and the Globular field (C) for the zone 14.

As definite formulas for the computation of the three curvature-components we have accepted:

$$\left.\begin{aligned} \frac{\partial \xi}{\partial x} &= 0.162\left\{3A + B\right\}\delta s + \Sigma\, 0.01 \cos^2\alpha\,(\triangle g - \triangle g_0) \\ \frac{\partial \eta}{\partial y} &= 0.162\left\{A + 3B\right\}\delta s + \Sigma\, 0.01 \sin^2\alpha\,(\triangle g - \triangle g_0) \\ -\frac{\partial \xi}{\partial y} &= -\frac{\partial \eta}{\partial x} = 0.324\, C\cdot\delta s + \Sigma\, 0.01 \sin\alpha\cos\alpha\,(\triangle g - \triangle g_0) \end{aligned}\right\} \quad \ldots \text{IV: 7}$$

Here the derivatives are in $''/10$ kms if δs is expressed in »mil» (1 mil = 10 kms) For the generalized A-field we have taken δs equal to 32 kms, thus integrating only for the zones 12, 13 and 14. Inside the zone 11 the parabolic representation has been used (see ch VI).

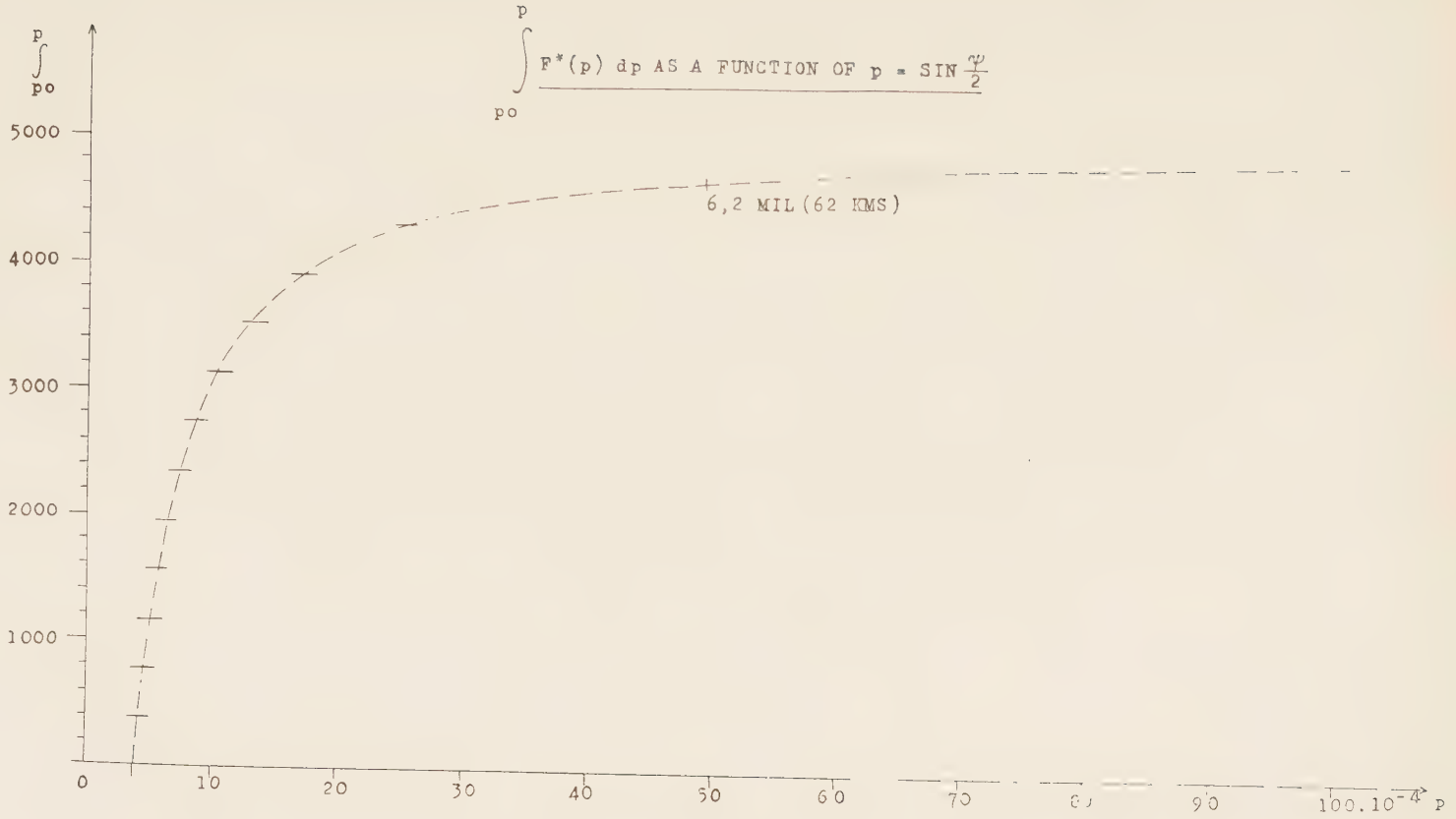

$\int_{po}^{p} F^*(p)\,dp$ AS A FUNCTION OF $p = \sin\frac{\varphi}{2}$
$\int_{po}^{p}$
5000
4000
3000
2000
1000
6,2 MIL (62 KMS)
0
10
20
30
40
50
60
70
80
90
100.10^{-4} p

Fig. 9 a.

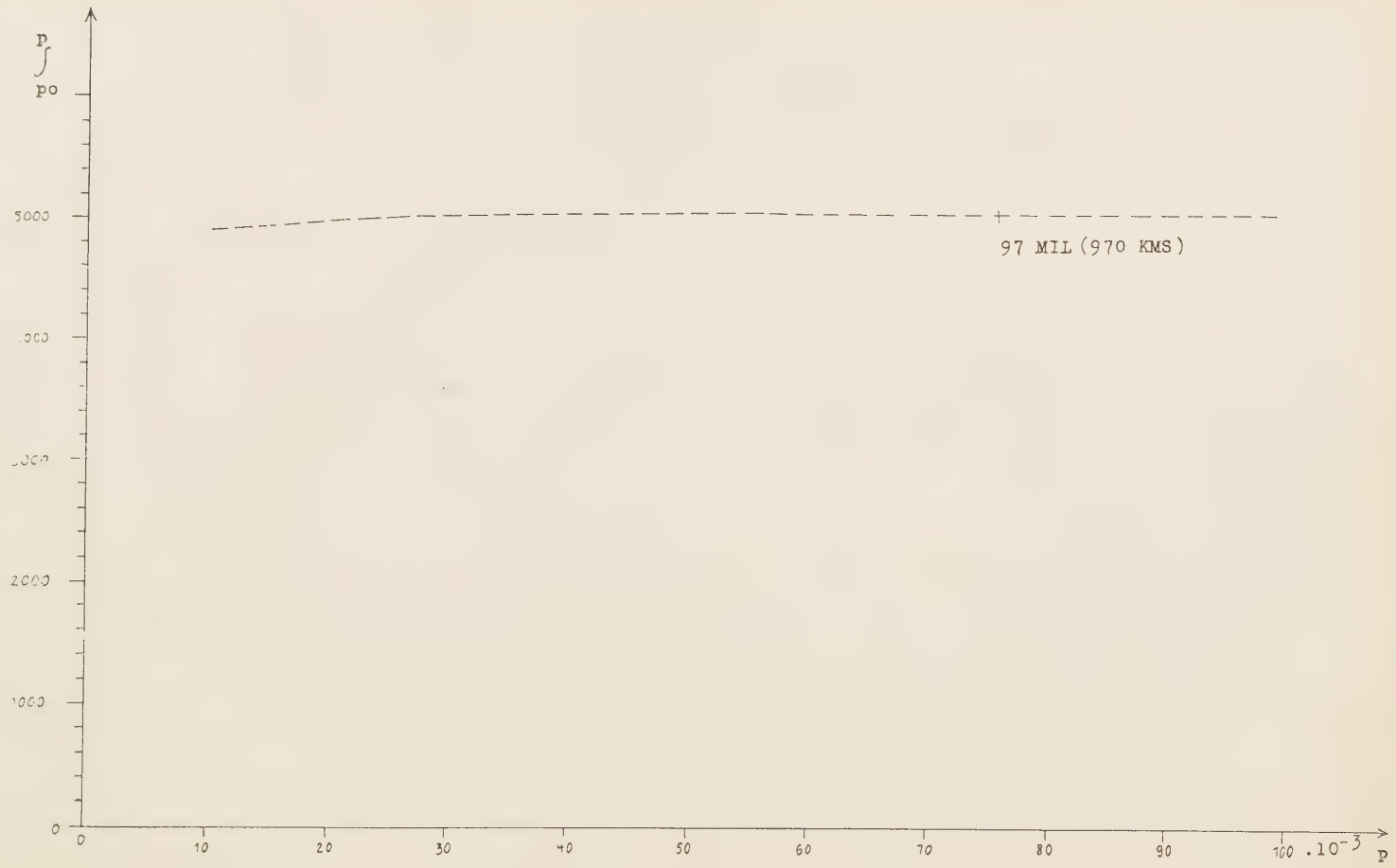

p
∫
po
5000
2000
1000
0
0
10
20
30
40
50
60
70
80
90
100 .10⁻³
p
97 MIL (970 KMS)

Fig. 9 b.

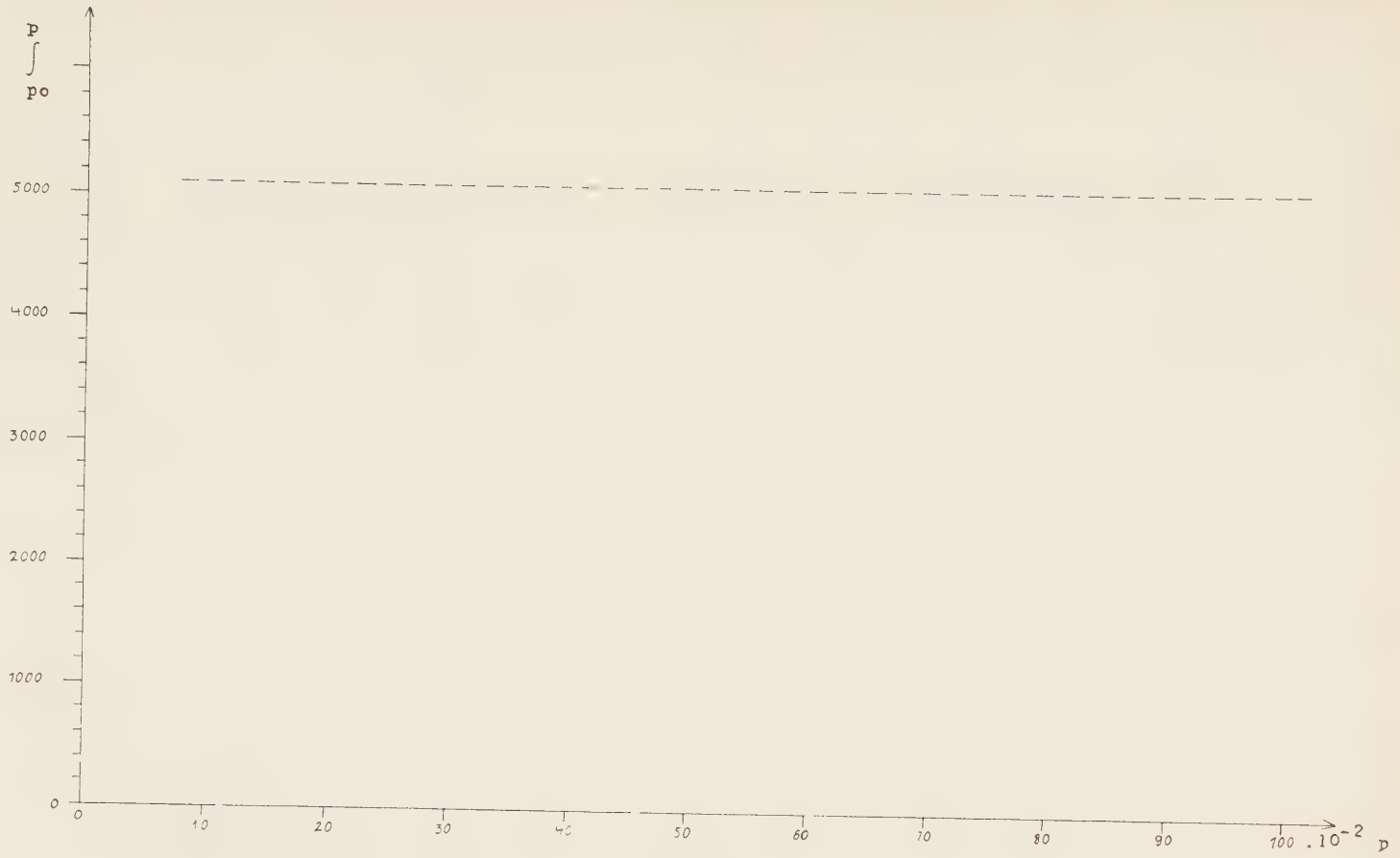

p
∫
po
5000
4000
3000
2000
1000
0
0
10
20
30
40
50
60
70
80
90
100 .10⁻²
p

Fig. 9 c.

V.

The astronomical deflectionpoints. Astrodeflections referred to the geoid.

The astronomical stations used in this investigation for the control of the gravimetrically determined differences of deflections are the following: *Östra Älvhöjden* (A_1), *Tornberget* (A_2), *Tomasboda* (A_3), *Norra Landsberget* (A_4) and *Gräsberget* (A_5). They are all first order geodetic points. A_1, A_2 and A_3 are furthermore Laplace-points of the first order. At A_4 and A_5 second order determinations of latitude and longitude have been carried out according to a modified »method of equal heights». (See further below). The deflections of the vertical have been calculated from the astronomical results and geodetical latitud- and longitud-values. The latter have been taken from the recent adjustment of the European first order triangulations (Northern Bloc), performed in the U.S.A. (bibl. 5). We obtain the following results of ξ_a and η_a (denoting them by ξ'_a and η'_a before the »sea-levelcorrection» has been added).

Station	Lat.	Long. E. Gr.	ξ'_a ″	η'_a ″	H mts	Astr. obs. in the year
A_1	60°00.′5	14°48.′0	— 17.8	+ 8.5	430	1937
A_2	60°00.′4	16°26.′2	— 8.9	+ 21.3	140	1935
A_3	59°23.′1	14°58.′4	— 2.2	+ 21.3	295	1936
A_4	59°55.′2	15°55.′5	— 13.8	+ 18.8	220	1952
A_5	59°57.′0	15°19.′3	— 19.2	+ 14.6	370	1952

The utilized astronomical latitude—determinations have been referred to the Fk 3-system and both latitudes and longitudes are corrected for the variation of the pole. The astronomical longitudes included in η'_a are calculated according to the observed differences from the accepted absolute value of the Swedish reference station at *Lovö* (bibl. 14). All longitudes are reckoned positive *eastwards*. H is the height above sea-level. For A_1, A_2 and A_3 no account has been taken of the η_a-value, obtained by the azimuthal observations here. The observational mean errors at the last mentioned stations correspond to an average mean error in ξ'_a and η'_a of about $\pm$ 0″.1 and $\pm$ 0″.5 respectively. As regards the second order points A_4 and A_5, these mean errors are of the magnitude $\pm$ 1″.1 and $\pm$ 1″.3 (see below).

The observations at A_4 and A_5 are carried out by a Wild T 3-theodolite according to the »method of equal heights» but allowing for a variation of 5° (amplitude) in height round the »almukantarat». One single determination of latitude and longitude consists in observing a group of three stars at an elevation of about 60°, 15 vertical

transits being taken for each star. At every transit (for slow stars pointing observation has been used) the chronometertime and vertical circle-reading has been noted. To obtain the best result from this observational arrangement (as regards both latitude and longitude) the latitude-star should be observed at an astronomical azimuth of about 15° *W* or *E*, and the two other stars in such directions that »the 120°-condition» of the method of equal heights will be fulfilled. The time signals have been observed with auditive coincidences from a short wave radio receiver (BC 348). The three-star groups have been chosen symmetrically on both sides of a time signal, the longitude star (azimuth 255° *W* or *E*) being taken less than 30 minutes from this signal. Beside ordinary signals, transmitted on short wave (Rugby, Moskva, Pontoise), the american transmission *WWV* has been used to interpolate chronometercorrections for shorter intervals. All time signals used in calculating the longitudes of A_4 and A_5 (see above) have been corrected to the »heure demidéfinitive» in the Bulletin Horaire.

The moments of transit for a star have been recorded with a registering watch with two hands, which will permit continuous observations, connecting the watch to the chronometer only once per star. To obtain the acceleration or retardation of the watch during the star-observation, a connection has been made after the last transit, too.

To have the variation of the »zenith» of the vertical circle under control, observations of Polaris have been made before and after the observation of each star.

A theoretical investigation (in 1951) af all known sources of errors, also with respect to those of systematic character made it probable that we could reach an accuracy by observing *12 groups*, which corresponds to a mean accidental error of about $\pm$ 0.″8 in ξ and η, the systematic error being even less. Calculations of the observations from 1952 show a somewhat greater mean accidental error. For five stations (including our deflection points A_4 and A_5) these mean errors varies from $\pm$ 0.″8 to $\pm$ 1.″4 in ξ and from $\pm$ 1.″0 to $\pm$ 1.″7 in η.

In order to be able to connect the above-mentioned second order determinations to the net of first order points, observations have also been carried out at Laplace stations. The observed *differences* in longitude between such stations agree satisfactorily with the first order ones. The discrepancies are of the same order of magnitude as the mean accidental errors mentioned above. The same can be stated for the latitude determinations, when compared to the first order ones (given in Fk 3).

To obtain a program of observation (selection of stars) I have used the following simple graphical method: A map of the brighter stars (down to the 6th magnitude) has a scale of rectascension expressed in time, which permits the reading of the local sidereal time for meridian transits with an error of a few minutes. The situation of the celestial pole within the »almukantarat», which has been drawn on a transparent sheet of paper and constructed in the star-map projection, is known with satisfactory accuracy. The pole plotted on the paper shall be fixed at the centre of the star map. Rotating the paper round this fixed point until the almukantaratcurve coincides with a star, the L.S.T. for the almukantarat-transit can be read of at the intersection between the curve's central line and the rectascension scale. Almost every star selected in this manner can be found in the »Apparent places of fundamental stars» (Fk 3-system).

To obtain ξ_a and η_a referred to the cogeoid (geoid), we should apply the »sea-levelcorrections» to ξ_a and η_a. The second term in $\triangle\xi_B$ (see I: 6 a), which depends upon the divergence of the normal gravity field (field of the »theoretical earth) takes the values $-0.''2$, $-0.''1$, $-0.''1$, $-0.''1$ and $-0.''2$ at the stations A_1, A_2, A_3, A_4 and A_5 respectively. $\triangle\eta_B$ and the first term of $\triangle\xi_B$, due to the additional divergence of the Bouguer field have been evaluated in the following manner. From the b_m-field (see fig. 2 on p. 24) we note that the b-values in the neighbourhood of all our deflection stations are close to the value of the factor 0.11 in the normal Bouguer correction ($\varrho = 2.67$). Thus, assuming the correlation between Bouguer-anomaly and height to be practically 0 and the interpolated Bouguerfield in the vicinity of the station to be satisfactorily representative we obtain the following values of $\triangle\xi_B$ and $\triangle\eta_B$ for our five stations: A_1 ($-0.''1$; $0.''0$), A_2 ($0.''0$; $0.''0$), A_3 ($0.''0$; $+0.''1$), A_4 ($0.''0$; $0.''0$) and A_5 ($0.''0$; $0.''0$).

The effects of topography and its »reflection» $-\triangle_t + \triangle_i$ (see ch I) have not been calculated because detailed and accurate topographical maps are not yet available for the terrain close to all deflection stations.

With respect to the great mean error of the reduction down to the generalized cogeoid, which has been made possible by using our interpolation method also in the vicinity of the deflection stations, we may neglect here the above mentioned small sea-level corrections. Only in such cases where steep topographical contours occur close to the stations, the correction $-\triangle_t + \triangle_i$ may be required (see the example on p. 16). Such topographical features, however, are not on hand at any one of our deflection stations.

I have, therefore, used here the values ξ'_a and η'_a directly as referred to the cogeoid without adding any sea-level corrections. But when denser gravity measurements out to a radius of say at least 5 kms have been carried out (see the discussion of mean error of reduction in ch VI) all components of the sea-level corrections should of course be evaluated at the deflection stations to be sure that their omission does not give rise to greater error than that obtained in the reduction to the generalized geoid.

VI.

Geoidal structure from curvatures in the central part of area A.

The numerical integrations for curvature components have been carried out for 169 points over the accepted generalized cogeoidal field (see ch II). These equally spaced curvature points cover an area of about 20.000 sqkms containing five astronomical deflection points, described in ch V ($A_1 - A_5$).

I have integrated graphically for the gravimetrical deflection differences, taking profiles of the derivatives $\frac{\partial \xi}{\partial x}$, $\frac{\partial \eta}{\partial y}$ and $\frac{\partial \xi}{\partial y}$, plotted as curve charts. An example of the result obtained is shown in fig. 10, giving the ξ-variations over the investigated area. The deflections are relative and reckoned from the point of intersection between the 15° E-meridian and the 60° N-parallel.

By calculating the components of curvature, the parabolic representation of the generalized field has been used up to a radius of 32 kms from the curvature point, and the corners of the anomaly pentagon (see ch I) have been placed at the gravity centres of the five outer compartments, each one having an area equal to $\frac{1}{6}$th of the area of the 32 km circle.

Denoting the mean anomalies at the centres and the corners of the pentagon by f_0, f_1, f_2, f_3, f_4 and f_5, f_1 having the azimuth 360°, we obtain the contributions $\left(\frac{\partial \xi}{\partial x}\right)_0$, $\left(\frac{\partial \eta}{\partial y}\right)_0$ and $\left(\frac{\partial \xi}{\partial x}\right)_0$ in the parabolic area $f = Ax^2 + By^2 + 2Cxy + Dx + Ey + f_0$ equal to

$$\left(\frac{\partial \xi}{\partial x}\right)_0 = -0.080\,(f_1 + f_2) - 0.012\,(f_3 + f_5) - 0.123\,f_4 + 0.307\,f_0$$

$$\left(\frac{\partial \eta}{\partial y}\right)_0 = -0.042\,(f_1 + f_2) - 0.112\,(f_3 + f_5) + 0.307\,f_0$$

$$\left(\frac{\partial \xi}{\partial y}\right)_0 = -0.058\,(f_2 - f_1) - 0.036\,(f_5 - f_3) = \left(\frac{\partial \eta}{\partial x}\right)_0$$

the derivatives being expressed in $''/10$ kms, f in mgals.

Adding the constant terms $-0.021\,f_0\,\Sigma\cos^2\alpha$, $-0.021\,f_0\,\Sigma\sin^2\alpha$ and $-0.021\,f_0\,\Sigma\sin\alpha\cos\alpha\,(=0)$, included in the total contribution from the integration zones 12, 13 and 14, we obtain the following expressions (contribution I)

$$\left(\frac{\partial \xi}{\partial x}\right)_1 = 0.519\, f_0 - 0.080\,(f_1 + f_2) - 0.012\,(f_3 + f_5) - 0.123\, f_4$$
$$\left(\frac{\partial \eta}{\partial y}\right)_1 = 0.519\, f_0 - 0.042\,(f_1 + f_2) - 0.112\,(f_3 + f_5) \qquad \text{VI : 1}$$
$$\left(\frac{\partial \xi}{\partial y}\right)_1 = 0.058\,(f_1 - f_2) + 0.036\,(f_3 - f_5) = \left(\frac{\partial \eta}{\partial x}\right)_1$$

The contributions from the zones 12, 13 a and 13 b have been taken from the A-field, the contributions from the parts 13 c, 13 d and 13 e, on the other hand, from the *B*-field. For 13 e this contribution has been found to be constant for all points within the investigated area and the same holds good for the contribution of zone 14 (taken from the *C*-field). For 13 d a graphical interpolation chart has been utilized.

To evaluate the accuracy obtained in the calculated curvature components with respect to the curvatures of the »true» field, generalized in the same manner, we may use the mean errors in the tables and figures on pages 28, 35, 36, 40 (ch II). We have, for instance, the following sums of the squares of error for $\left(\frac{\partial \xi}{\partial x}\right)$.

A-field	
Parabolic area + const. term	0.3267
zone 12	0.0014
zone 13 a + 13 b	0.0001
B-field	
zone 13 c	0.0002
zone 13 d	0.0000
zone 13 e	0.0000
C-field	
zone 14	0.0000

As a total, we obtain a mean error in the $\frac{\partial \xi}{\partial x}$ *— and* $\frac{\partial \eta}{\partial y}$ *— components of curvature equal to about* $\pm$ 0.″6/10 kms.

It will be seen that the greatest part of the variance corresponds to the mean error of representation in f_0. Thus we may regard our generalized curvature components $\frac{\partial \xi}{\partial x}$ and $\frac{\partial \eta}{\partial y}$ for points at greater distances than, say, 25 kms as nearly independent. This corresponds to an approximate mean error in differences of ξ along meridians and of η along parallels of $\pm$ 1″ for points at the aforesaid distance. The propagation of mean errors for multiples of this distance takes place at a somewhat more rapid rate than that according to the formula $K \sqrt{S}$. In fact, for a distance of (S) 50 kms, the mean error of the deflectiondifferences (along parallels and meridians) will be about $\pm$ 1.″8, and for a distance of 100 kms + 2,″4 etc. As regards $\frac{\partial \xi}{\partial y}$ and $\frac{\partial \eta}{\partial x}$ the mean error is negligible

when compared to the mean error of the principle components $\frac{\partial \xi}{\partial x}$ and $\frac{\partial \eta}{\partial y}$ (the former do not depend upon f_0). When calculating the components of curvature it was not necessary to carry out the computations for all points according to the integrational part of the formula IV: 7 (p. 54). For 28 equally spaced points I have plotted the contributions from all compartments outside the 32 km-circle as curves. From these curves the correction to the »parabolic» result can be taken and thus the absolute curvature-components will be obtained. From ξ- and η- profiles evaluated along the meridian and parallel through the chosen reference point (15° *E*, 60° *N*), I have determined the ξ-field and η-field in the following manner:

To obtain the former field, meridian profiles of ξ have been calculated and smoothed out, account being taken of the mean error in the curvature component $\frac{\partial \xi}{\partial x}$ (see above). We can check this preliminary ξ-field, when we compare its east-west profiles with the same profiles deduced from integrations over $\frac{\partial \xi}{\partial y}$. Within 150 kms distance from the central point, the differences of ξ ordinarily do not exceed 4″. Taking the mean as a definite ξ-value, we accept the simple principle for adjustment: *The two shortest and equally weighted rectangular paths of integration must give the same result for each point.* In the same way the η-field has been constructed.

To check our ξ-and η-chart we have to compare it with deflection differences between the astronomical stations $A_1 - A_5$. In order to make the latter comparable to the gravimetrically calculated deflections we have to correct them for the difference between the real cogeoidal field and the accepted generalized field, on which the gravimetrical computations have been based. This reduction has been made in the following manner.

Considering the b_m-values, taken from the b_m-chart, to be sufficiently representative also for the neighbourhood of the deflection stations, I have constructed a cogeoidal field from mean heights (interpreted on a topographical map of scale 1: 100 000) over a circle of 9.3 km radius. The *a*-values have also been regarded as sufficiently accurate for our purpose. The circle represents the inner field of 5 km radius + the deflection zones 1 and 2, and the parabolic representation has been used over the circle. The contribution to the reduction (generalized deflection − real deflection) from this parabolic field is

$$\left.\begin{aligned} \delta\xi_0 &= 0.150\,(\triangle f_1 + f_2) - 0.057\,(\triangle f_3 + \triangle f_5) - 0.186\,\triangle f_4 \\ \delta\eta_0 &= -0.109\,(\triangle f_2 - \triangle f_1)\; 0.177\,(\triangle f_5 - \triangle f_3) \end{aligned}\right| \quad \ldots\ldots \text{VI: 4.}$$

$\triangle f$ being the difference between the mean anomalies of the generalized field and the real field. The mean error of these *reductions* will depend solely upon the mean error of f in the *real* field. Assuming this mean error to be calculable from M_0 by Gauss' law (see page 28) we have the mean errors of $\delta\xi_0$ and $\delta\eta_0$ equal to about $\pm$ 2.″0. In order to reduce this error, it would be necessary to establish a greater gravimetrical

station density in the vicinity of the deflection stations. Wideland (bibl. 23) has carried out such dense measurements at A_1, A_2 and A_3. They have, however, not been utilized here.

In relation to the error of the contribution from the vicinity of the deflection stations, the error of the integrational part in the reduction may be neglected. I have integrated from the deflection zone 3 to the deflection zone 8 (9.3 kms to 58.6 kms) regarding the mean anomalies of the deflection compartments in the real and generalized field as identical beyond the distance 58.6 kms.

The results of the investigation for relative deflections within the central part of area A (ξ- and η-charts) can now be compared with the astronomic-geodetic deflections, reduced to the generalized geoid.

The reduced astrodeflections are:

Point	ξ_a "	η_a "
A_1	— 13	+ 8
A_2	— 9	+ 22
A_3	— 3	+ 16
A_4	— 15	+ 18
A_5	— 18	+ 14

and the gravimetrical deflections at the same points

Point	ξ_g "	η_g "
A_1	$0 + k_1$	$0 + k_2$
A_2	$+ 4 + k_1$	$+ 8 + k_2$
A_3	$+ 15 + k_1$	$+ 3 + k_2$
A_4	$+ 2 + k_1$	$+ 9 + k_2$
A_5	$- 1 + k_1$	$+ 4 + k_2$

From the tables we obtain the most probable values of k_1 and k_2

$$k_1 = - 15'' \pm 1''$$
$$k_2 = + 11'' \pm 2''$$

The mean error of a ξ-difference taken from the ξ-chart will be $\pm 2.''7 \sqrt{2} (= \pm 3.''8)$, the corresponding mean error for η being $\pm 2.''4 \sqrt{2} (= \pm 3.''4)$. Thiese errors chiefly depend upon the uncertainty of the reduction, that is upon the lack of gravity material in the vicinity of the deflection stations.

We may also compare the deflection-differences on our generalized geoid with those of an isostatic cogeoid, which has been studied by Wideland as regards both astro-deflections and gravimetrical deflections (bibl. 20, 23).

The table below shows, that (for A_1, A_2 and A_3) our generalized geoid coincides well with the isostatic cogeoid, the discrepancies being even smaller than those obtained from Wideland's gravimetrical deflections.

Point	ξ_a" (isost.)	ξ_g" (gen.)	ξ_g" (Wid.)	$\triangle$ (gen.)	$\triangle$ (Wid.)	η_a" (isost.)	η_g" (gen)	η_g" (Wid.)	$\triangle$ (gen.)	$\triangle$ (Wid.)
A_1	0	0	0			0	0	0		
A_2	+ 6	+ 4	+ 5	+ 2	+ 1	+ 13	+ 8	+ 4	+ 5	+ 9
A_3	+ 16	+ 16	+ 16	0	+ 1	+ 7	+ 3	+ 4	+ 4	+ 3

At last, a comparison shall be made with the results, obtained for deflection-differences by Rice (bibl. 16) in 1951. The mean distance between the tabulated points is about 100 kms.

	$\triangle \xi_a$"	$\triangle \xi_g$"	$\delta \triangle \xi$	$\triangle \eta_a$"	$\triangle \eta_g$"	$\delta \triangle \eta$"
Brooks—Burns	— 0.95	— 0.33	+ 0.6	— 2.38	— 2.60	+ 0.2
Ecore—Burns	— 0.63	— 0.14	+ 0.5	+ 0.69	+ 0.18	— 0.5
Roby—Bartley	+ 1.61	+ 1.08	— 0.5	— 0.79	+ 0.73	+ 1.5
Sears—Lacasa	+ 3.22	+ 3.47	+ 0.3	— 1.62	— 1.91	— 0.3
Roby—Polk	+ 5.89	+ 5.50	— 0.4	— 3.07	— 3.83	— 0.7
Lacara—Polk	+ 7.77	+ 7.27	— 0.5	+ 0.69	— 0.18	— 0.9

which has a formal mean error in $\triangle \xi_a$" equal to $\sim \pm 2$" and in $\triangle \eta_g$ equal to $\sim \pm 3$".

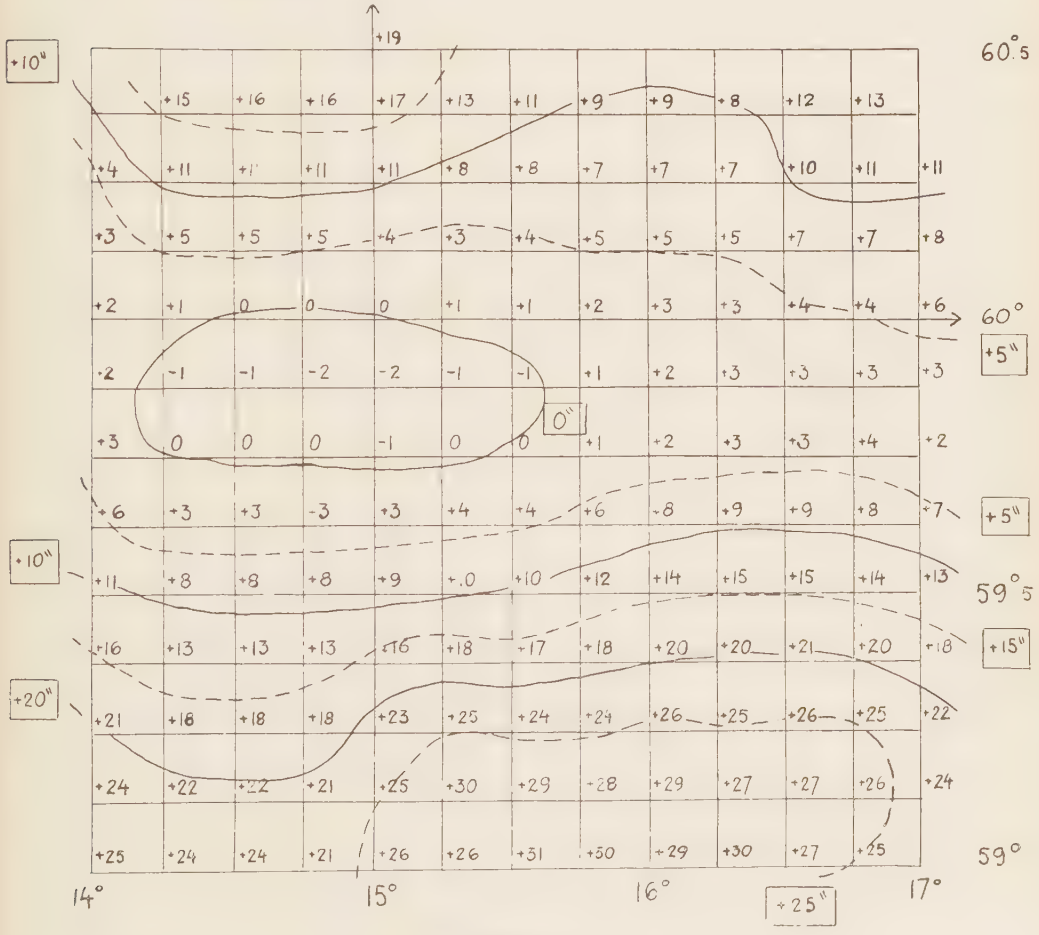


Fig. 11.

VII.

A determination of the absolute deflection at Östra Älvhöjden.

With the aid of the anomaly-charts over the *A*-, *B*- and *C*-fields, it will now be possible to calculate the absolute values of ξ and η at the first deflection-station A_1. I believe this point to be the most favorable one for such computations in our country, perhaps in the whole Scandinavia, the gravity net being relatively dense and homogenous in the x- and y-directions up to a radius of 500 kms.

From the investigation of mean error of representation in ch II, we conclude that the total mean error in ξ and η ought to be of the magnitude $\pm 4''$. The contribution to this mean error from the different fields of integration are the following (square sums)

	ξ	η	zones
A-field	0.357	0.357	1—12
B-field	1.145	1.960	13—18
C-field	14.008	11.397	19—36

With regard to the field within the circle of radius 9.3 kms (up to zone 3), I have estimated the mean error from the error of representation in mean anomalies, taken (as by the reduction in ch VI) from the combined b_m- and a_m-charts, using mean heights from a larger scale topographical map (1: 100.000). The mean error from the parabolic field of 5 kms radius has been calculated according to the formulas for $\delta \xi_0$ and $\delta \eta_0$ (see p. 63).

The results for ξ_g and η_g are the following:

Östra Älvhöjden (on the geoid).

$\xi_g = -8'' \pm 4''$

$\eta_g = +12'' \pm 4''$

the great mean error depending mainly upon the uncertainty of the Globular field and the field in the vicinity of the station. η_g is consistent with the astronomic-geodetic value $+8.''5$, the difference being smaller than the mean error. But with regard to ξ_g the difference seems to indicate a real correction to the astronomic-geodetic value, that is, also to the value of ξ at Potsdam accepted for the adjustment of the European triangulation network. The correction would be $+10'' \pm 4''$, a value, which agrees well with the the determinations of Ledersteger (bibl. 12) and Wolf (bibl. 24).

VIII.

Some remarks concerning the geoid-determination over larger areas.

In order to determine the geoid over the whole country by the aid of the method method indicated in this paper we have to orientate the separate »curvature» areas in the same reference system. This may be done in the following simple manner: Astronomical stations are placed at the corners of squares of the size, say, 150 × 150 sqkms (for our purpose the aforesaid second order determinations of longitude and latitude will suffice). By calculating the ξ_g-and η_g-profiles along the sides of the squares, the discrepancies between gravimetrical deflection-differences and the corresponding astronomic-geodetic differences can be reduced to zero and a correction applied to each curvature point (proportional to the distance from a corner) along the sides of the square. The gravimetrical profiles of ξ and η are now adjusted upon the known differences of astrodeflections. There profiles may be considered as definite at the remaining deformation of all computed ξ-and η-profiles within the square. Taking at first all profiles in the x-direction, we may correct them (in the same manner as above) in order to make the differences in deflections between the ends of the profiles coincide with the differences obtained from the adjustment of the opposite side-profiles. After having computed the corresponding corrections in the y-direction we have for each curvature-point two values of ξ and η. The means of these values should be regarded as definite and the ξ- and η-fields can be plotted as curves. This adjustment is not, of course, fully consistent with the law for propagation of errors in the graphical integrations. It presumes a linear law, which will be inaccurate at greater distances. But I think it will be good enough by evaluating the undulations of our generalized geoid within the separate »curvature»-areas. In order to obtain this geoid for the whole country referred to the international ellipsoid, we must know the values of ξ, η and N at an initial point. But usually, this absolute orientation is not the most important thing asked for at geoidal investigations. It would suffice to know the geoid of the country as one surface, the undulations of which are correct over short distances as well as along extended profiles. The orientation could be carried out when these values of ξ, η and N are known with sufficient accuracy. Only is there a need for an approximate knowledge of the absolute elements at the initial station, in order to be able to free the astronomic-geodetic deflection-differences on the international ellipsoid from that systematical part, which depends upon the distance. For our country with Östra Älvhöjden as initial station the error of the accepted values of ξ_0, η_0 and N_0 (— 18", + 9",0 mts) will not influence our relative astronomic-geodetic deflections by more than 0."2 at the greatest distances from the initial station, the errors of ξ_0, η_0 and N_0 being of the magnitudes — 10", + 2", 20 mts. (see ch VII, bibl. 19).

The investigation of the geoid by the aid of curvatures cannot be carried out in the neighbourhood of »empty» gravity regions, as for instance along our coasts and along some extended mountainterritories. In the first case we must either make rather dense gravity measurements at sea or we have to determine the geoidal undulations by astronomical observations alone. We may assume that the gravity exploration of the Baltic will probably not take place in the next future. I, therefore, believe that the best planning for geoidal studies in the coast-area will be to begin now to determine astronomical deflection points with a proposed average station density of about 1 p./400 sqkms. This density will not give as many details of the geoidal surface as does the gravity field within the investigated area, but the method will of course be more accurate for the determination of single deflections.

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